Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.
Number Sense and Numeration, Grades 4 to 6

Volume 1
The Big Ideas

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6
INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of The Ontario Curriculum, Grades 1–8: Mathematics, 2005. This guide provides teachers with practical applications of the principles and theories that are elaborated in A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006.

The guide comprises the following volumes:
- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

This first volume of the guide provides a detailed discussion of the five “big ideas”, or major mathematical themes, in Number Sense and Numeration. The guide emphasizes the importance of focusing on the big ideas in mathematical instruction to achieve the goal of helping students gain a deeper understanding of mathematical concepts.

Volumes 2 to 6 of the guide focus on the important curriculum topics of addition and subtraction, multiplication, division, fractions, and decimal numbers, respectively. Each of these volumes provides:
- a description of the characteristics of junior learners and the implications those characteristics have for instruction;
- a discussion of mathematical models and instructional strategies that have proved effective in helping students understand the mathematical concepts related to the topic;
- sample learning activities, for Grades 4, 5, and 6, that illustrate how a learning activity can be designed to:
  - focus on an important curriculum topic;
  - involve students in applying the seven mathematical processes described in the mathematics curriculum document;
  - develop understanding of the big ideas in Number Sense and Numeration.
Each of the volumes also contains a list of the references cited throughout the guide. A glossary that includes mathematical and pedagogical terms used throughout the six volumes is included at the end of Volume 1.

The content of all six volumes of the guide is supported by "eLearning modules" that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities in Volumes 2 through 6.

**Working Towards Equitable Outcomes for Diverse Students**

All students, regardless of their socioeconomic, ethnocultural, or linguistic background, must have opportunities to learn and to grow, both cognitively and socially. When students see themselves reflected in what they are learning, and when they feel secure in their learning environment, their true potential will be reflected in their achievement. A commitment to equity and inclusive instruction in Ontario classrooms is therefore critical to enabling all students to succeed in school and, consequently, to become productive and contributing members of society.

To create the right conditions for learning, teachers must take care to avoid all forms of bias and stereotyping in resources and learning activities, which can quickly alienate students and limit their ability to learn. Teachers should be aware of the need to provide a variety of experiences and multiple perspectives, so that the diversity of the class is recognized and all students feel respected and valued. Learning activities and resources for teaching mathematics should be inclusive in nature, providing examples and illustrations and using approaches that reflect the range of experiences of students with diverse backgrounds, abilities, interests, and learning styles.

The following are some strategies for creating a learning environment that recognizes and respects the diversity of students, and allows them to participate fully in the learning experience:

- providing mathematics problems with contexts that are meaningful to all students (e.g., problems that reflect students' interests, home-life experiences, and cultural backgrounds);
- using mathematics examples that reflect diverse ethnocultural groups, including Aboriginal peoples;
- using children's literature that reflects various cultures and customs;
- respecting customs and adjusting teaching strategies, as necessary. For example, a student may come from a culture in which it is considered inappropriate for a child to ask for help, express opinions openly, or make direct eye contact with an adult;
- considering the appropriateness of references to holidays, celebrations, and traditions;
- providing clarification if the context of a learning activity is unfamiliar to students (e.g., describing or showing a food item that may be new to some students);
- evaluating the content of mathematics textbooks, children's literature, and supplementary materials for cultural bias;
• designing learning and assessment activities that allow students with various learning styles (e.g., auditory, visual, tactile/kinaesthetic) to participate meaningfully;
• providing opportunities for students to work both independently and with others;
• providing opportunities for students to communicate orally and in writing in their home language (e.g., pairing English language learners with a first-language peer who also speaks English);
• using diagrams, pictures, manipulatives, and gestures to clarify mathematical vocabulary that may be new to English language learners.

For a full discussion of equity and diversity in the classroom, as well as a detailed checklist for providing inclusive mathematics instruction, see pp. 34–40 in Volume 1 of A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006.

An important aspect of inclusive instruction is accommodating students with special education needs. The following section discusses accommodations and modifications as they relate to mathematics instruction.

**Accommodations and Modifications**

The learning activities in Volumes 2 through 6 of this guide have been designed for students with a range of learning needs. Instructional and assessment tasks are open-ended, allowing most students to participate fully in learning experiences. In some cases, individual students may require accommodations and/or modifications, in accordance with their Individual Education Plan (IEP), to support their participation in learning activities.

**Providing accommodations**

Students may require accommodations, including special strategies, support, and/or equipment to allow them to participate in learning activities. There are three types of accommodations:

- **Instructional accommodations** are adjustments in teaching strategies, including styles of presentation, methods of organization, or the use of technology or multimedia.
- **Environmental accommodations** are supports or changes that the student may require in the physical environment of the classroom and/or the school, such as preferential seating or special lighting.
- **Assessment accommodations** are adjustments in assessment activities and methods that enable the student to demonstrate learning, such as allowing additional time to complete tasks or permitting oral responses to test questions.

Some of the ways in which teachers can provide accommodations with respect to mathematics learning activities are listed in the table on pp. 8–9.

The term accommodations is used to refer to the special teaching and assessment strategies, human supports, and/or individualized equipment required to enable a student to learn and to demonstrate learning. Accommodations do not alter the provincial curriculum expectations for the grade.

Modifications are changes made in the age-appropriate grade-level expectations for a subject . . . in order to meet a student’s learning needs. These changes may involve developing expectations that reflect knowledge and skills required in the curriculum for a different grade level and/or increasing or decreasing the number and/or complexity of the regular grade-level curriculum expectations.

**Instructional Accommodations**

- Vary instructional strategies, using different manipulatives, examples, and visuals (e.g., concrete materials, pictures, diagrams) as necessary to aid understanding.
- Rephrase information and instructions to make them simpler and clearer.
- Use non-verbal signals and gesture cues to convey information.
- Teach mathematical vocabulary explicitly.
- Have students work with a peer.
- Structure activities by breaking them into smaller steps.
- Model concepts using concrete materials, and encourage students to use them when learning concepts or working on problems.
- Have students use calculators and/or addition and multiplication grids for computations.
- Format worksheets so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Encourage students to use graphic organizers and graph paper to organize ideas and written work.
- Provide augmentative and alternative communications systems.
- Provide assistive technology, such as text-to-speech software.
- Provide time-management aids (e.g., checklists).
- Encourage students to verbalize as they work on mathematics problems.
- Provide access to computers.
- Reduce the number of tasks to be completed.
- Provide extra time to complete tasks.

**Environmental Accommodations**

- Provide an alternative workspace.
- Seat students strategically (e.g., near the front of the room; close to the teacher in group settings; with a classmate who can help them).
- Reduce visual distractions.
- Minimize background noise.
- Provide a quiet setting.
- Provide headphones to reduce audio distractions.
- Provide special lighting.
- Provide assistive devices or adaptive equipment.
Assessment Accommodations

- Have students demonstrate understanding using concrete materials or orally rather than in written form.
- Have students record oral responses on audiotape.
- Have students’ responses on written tasks recorded by a scribe.
- Provide assistive technology, such as speech-to-text software.
- Provide an alternative setting.
- Provide assistive devices or adaptive equipment.
- Provide augmentative and alternative communications systems.
- Format tests so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Provide access to computers.
- Provide access to calculators and/or addition and multiplication grids.
- Provide visual cues (e.g., posters).
- Provide extra time to complete problems or tasks or answer questions.
- Reduce the number of tasks used to assess a concept or skill.

Modifying curriculum expectations

Students who have an IEP may require modified expectations, which differ from the regular grade-level curriculum expectations. When developing modified expectations, teachers make important decisions regarding the concepts and skills that students need to learn.

Most of the learning activities in this document can be adapted for students who require modified expectations. The table on p. 10 provides examples of how a teacher could deliver learning activities that incorporate individual students’ modified expectations.

It is important to note that some students may require both accommodations and modified expectations.
### Modified Program

<table>
<thead>
<tr>
<th>Modified Program</th>
<th>What It Means</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified learning expectations, same activity, same materials</td>
<td>The student with modified expectations works on the same or a similar activity, using the same materials.</td>
<td>The learning activity involves solving a problem that calls for the addition of decimal numbers to thousandths using concrete materials (e.g., base ten materials). Students with modified expectations solve a similar problem that involves the addition of decimal numbers to tenths.</td>
</tr>
<tr>
<td>Modified learning expectations, same activity, different materials</td>
<td>The student with modified expectations engages in the same activity, but uses different materials that enable him/her to remain an equal participant in the activity.</td>
<td>The activity involves ordering fractions on a number line with unlike denominators using Cuisenaire rods. Students with modified expectations may order fractions with like denominators on a number line using fraction circles.</td>
</tr>
<tr>
<td>Modified learning expectations, different activity, different materials</td>
<td>Students with modified expectations participate in different activities.</td>
<td>Students with modified expectations work on a fraction activities that reflect their learning expectations, using a variety of concrete materials.</td>
</tr>
</tbody>
</table>

(Adapted from Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6, p. 119)
THE “BIG IDEAS” IN NUMBER SENSE AND NUMERATION

The big ideas in Number Sense and Numeration in Grades 4 to 6 are as follows:
- quantity
- operational sense
- relationships
- representation
- proportional reasoning

The curriculum expectations outlined in the Number Sense and Numeration strand for each grade in The Ontario Curriculum, Grades 1–8: Mathematics, 2005 are organized around these big ideas.

The discussion of each big idea in this volume provides:
- an overview, which includes a general discussion of the big idea in the junior grades, along with an explanation of some key concepts inherent in the big idea;
- descriptions of the characteristics of learning that are evident among students in Grades 4–6 who have a strong understanding of the big idea (or, in the case of proportional reasoning, a developing understanding);
- descriptions of general instructional strategies that help students develop a strong understanding of the big idea.

In developing a mathematics program, it is important to concentrate on the big ideas and on the important knowledge and skills that relate to those big ideas. Programs that are organized around big ideas and focus on problem solving provide cohesive learning opportunities that allow students to explore mathematical concepts in depth. An emphasis on big ideas contributes to the main goal of mathematics instruction – to help students gain a deeper understanding of mathematical concepts.

1. In the mathematics curriculum document, the specific expectations are grouped under subheadings that reflect particular aspects of the knowledge and skills students are expected to learn. For example, the subheadings in the Number Sense and Numeration strand for Grade 5 include “Quantity Relationships”, “Operational Sense”, and “Proportional Reasoning”. The subheadings for the strand reflect the big ideas identified in A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 3 – Number Sense and Numeration, 2003, and those identified here for Grades 4 to 6.
Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004 outlines components of effective mathematics instruction, including a focus on big ideas in student learning:

“When students construct a big idea, it is big because they make connections that allow them to use mathematics more effectively and powerfully. The big ideas are also critical leaps for students who are developing mathematical concepts and abilities.”

(Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004, p. 19)

Students are better able to see the connections in mathematics, and thus to learn mathematics, when it is organized in big, coherent “chunks”. In organizing a mathematics program, teachers should concentrate on the big ideas in mathematics and view the expectations in the curriculum policy documents for Grades 4 to 6 as being clustered around those big ideas.

The clustering of expectations around big ideas provides a focus for student learning and for teacher professional development in mathematics. Teachers will find that investigating and discussing effective teaching strategies for a big idea is much more valuable than trying to determine specific strategies and approaches to help students achieve individual expectations. In fact, using big ideas as a focus helps teachers to see that the concepts presented in the curriculum expectations should not be taught as isolated bits of information but rather as a network of interrelated concepts.

In building a program, teachers need a sound understanding of the key mathematical concepts for their students’ grade level, as well as an understanding of how those concepts connect with students’ prior and future learning (Ma, 1999). Such knowledge includes an understanding of the “conceptual structure and basic attitudes of mathematics inherent in the elementary curriculum” (p. xxiv), as well as an understanding of how best to teach the concepts to students. Concentrating on developing this knowledge will enhance effective teaching and provide teachers with the tools to differentiate instruction.

Focusing on the big ideas provides teachers with a global view of the concepts represented in the strand. The big ideas also act as a “lens” for:

- making instructional decisions (e.g., choosing an emphasis for a lesson or set of lessons);
- identifying prior learning;
- looking at students’ thinking and understanding in relation to the mathematical concepts addressed in the curriculum (e.g., making note of the ways in which a student solves a division problem);
- collecting observations and making anecdotal records;
- providing feedback to students;
• determining next steps;
• communicating concepts and providing feedback on students’ achievement to parents² (e.g., in report card comments).

All learning, especially new learning, should be embedded in well-chosen contexts for learning – that is, contexts that are broad enough to allow students to investigate initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. Such rich contexts for learning open the door for students to see the “big ideas”, or key principles, of mathematics, such as pattern or relationship.

(Ontario Ministry of Education, 2005, p. 25)

² In this document, parent(s) refers to parent(s) and guardian(s).
Overview

Understanding the quantity represented by a number is an important aspect of having number sense. In the primary grades, students first explore quantities of numbers less than 10 (e.g., the meaning of “5”), and later learn to relate quantities to significant numbers, such as 10 and 100 (e.g., 12 is 2 more than 10; 20 is 2 tens; 98 is 2 less than 100). Towards the end of the primary grades, students gain an understanding of quantities associated with two- and three-digit whole numbers.

In the junior grades, students develop a sense of quantity of multidigit whole numbers, decimal numbers, fractions, and percents. The use of concrete and pictorial models (e.g., base ten blocks, fraction circles, 10 x 10 grids) is crucial in helping students develop an understanding of the “howmuchness” of different kinds of numbers, and provides experiences that allow students to interpret symbolic representations of numbers (e.g., understanding the quantity represented by “9/10”).

The following are key points that can be made about quantity in the junior grades:

- Having a sense of quantity involves understanding the “howmuchness” of whole numbers, decimal numbers, fractions, and percents.
- Experiences with numbers in meaningful contexts help to develop a sense of quantity.
- An understanding of quantity helps students estimate and reason with numbers.
- Quantity is important in understanding the effects of operations on numbers.

QUANTITY AS “HOWMUCHNESS”

For students in the junior grades, possessing number sense depends on an understanding of quantities represented by whole numbers, decimal numbers, fractions, and percents. It is important that students have many opportunities to represent numbers using concrete and
pictorial models, in order to gain a sense of number size. For example, experiences in using base ten blocks to represent multidigit whole numbers help students understand the base ten relationships in our number system, and the meaning of the digits in a number (e.g., in 3746 there are 3 thousands, 7 hundreds, 4 tens, 6 ones). Representing multidigit whole numbers also allows students to observe that place values increase by powers of 10 as they move to the left (ones, tens, hundreds, thousands, ten thousands, and so on).

Students learn that decimal numbers, as an extension of the base ten number system, can be used to represent quantities less than 1 (e.g., 0.2 represents 2 tenths; 0.54 represents 54 hundredths). Representing decimal numbers using materials (e.g., base ten blocks, fraction strips divided into tenths, 10 × 10 grids) helps students to visualize decimal quantities.

Representing fractions using a variety of models (e.g., fraction strips, fraction circles, grids) develops students’ ability to interpret fractional quantities. Providing opportunities to solve problems using concrete materials and drawings helps students to understand the quantity represented by fractions. Consider the following problem.

“Five children share 3 submarine sandwiches equally. How much will each child receive?”

**Fraction Strips Showing Five Equal Portions**

Each child receives \( \frac{3}{5} \) of a submarine sandwich.

Using concrete materials and drawings also helps students to observe the proximity of a fraction to the numerical benchmarks of 0, 1/2, and 1 whole (e.g., 5/6 is a quantity that is close to 1 whole; 3/8 represents an amount that is close to 1/2; 1/10 is close to 0).

**Fraction Circles Showing How Much of Each Whole Is Shaded**

\( \frac{5}{6} \) is close to 1.

\( \frac{3}{8} \) is close to \( \frac{1}{2} \).

\( \frac{1}{10} \) is close to 0.
Representing fractions as parts of wholes also helps students to understand concepts about the size of fractional parts – the greater the numeral in the denominator, the smaller the fractional parts. For example, a whole pizza divided into sixths yields bigger slices than a pizza of equal size divided into eighths.

In Grade 6, students investigate the meaning of percent and learn that percents, like decimal numbers and fractions, are used to represent quantities that are parts of a whole. Models, such as 10 × 10 grids, provide students with representations of percents and also demonstrate that quantities can be represented by different, but equivalent, number forms.

In the junior grades, students’ understanding of quantity develops not only through experiences with modelling numbers concretely and pictorially but also through opportunities to represent numbers more abstractly. In particular, number lines provide a model for representing number quantities. Although a number line does not show the quantities associated with different numbers, it does provide a tool by which students can represent the relationships between number quantities. For example, given an open number line with only 0 and 1 indicated,
students would need to reason about the relative quantities of numbers such as 1/4, 0.9, 45%, and their relationships to the benchmark numbers of 0, 1/2, and 1, in order to estimate their approximate positions on the line.

Open Number Line

EXPERIENCES WITH QUANTITY IN MEANINGFUL CONTEXTS

In the junior grades, there can be a tendency to focus on teaching computational procedures and to give less attention to the development of students’ comprehension of quantity. It is important to provide students with opportunities to make sense of number quantities by relating them to meaningful contexts. For example, students might:

• discuss the reasonableness of numbers in various situations (e.g., “Could 1000 people stand in our classroom? Could 1000 people stand in the gym?”);
• discuss how the same number can be used to express different notions of quantity (e.g., 58 students, 58 degrees, 58 kilometres, 58 minutes, 58 dollars, 58 hundredths);
• explore the magnitude of large and small numbers by placing them into meaningful contexts (e.g., to get an idea of how significantly greater 1 billion is than 1 million, students might explore these numbers in relation to time – 1 million seconds is a little more than 11 days, while 1 billion seconds is about 32 years!);
• solve numerical problems that are based on meaningful and interesting contexts;
• reflect on the size of an anticipated solution (e.g., “Will 5 × $175 be greater or less than $1000? How do you know?”);
• justify their solutions to problems involving numbers (e.g., “If you need 3/8 of a metre of ribbon to make a bow, how many bows can you make with 1 metre of ribbon?”).

QUANTITY AS IT RELATES TO ESTIMATING AND REASONING WITH NUMBERS

Having a sense of quantity is crucial in developing estimation skills. In the primary grades, students develop reasoning strategies to make appropriate estimates of quantities. For example, to estimate the number of marbles in a jar, students might think about what a group of 10 marbles looks like, and then imagine the number of groups of 10 in the large container. Students in the junior grades learn to apply similar reasoning to estimate larger quantities (e.g., imagining the size of a group of 20 students in order to estimate the number of students in a school assembly).

Students should have opportunities to observe quantities of 100, 500, and 1000, and to use these quantities to estimate larger amounts. Whole-class projects that involve collecting

Numbers are ideas – abstractions that apply to a broad range of real and imagined situations. (Kilpatrick, Swafford, & Findell, 2001, p. 72)
and keeping count of small objects (e.g., buttons, popcorn kernels, pennies) allow students to develop reference points or benchmarks for imagining large quantities. For example, “If a coffee cup holds 100 marbles, what size container would hold 1000 marbles? 10 000 marbles? 100 000 marbles?”

Students should also develop mental images of fractional amounts and use these images to estimate quantities. Learning activities in which students model fractions, decimal numbers, and percents using concrete materials (e.g., fraction circles, fraction strips, base ten blocks) and diagrams (e.g., number lines, 10 × 10 grids) allow students to visualize common fractional quantities, such as 1/4, 0.75, and 50%. Students can use these images to estimate other fractional amounts (e.g., the jar is about 1/4 full of paint; a little more than 50% of the magazine is filled with advertisements).

QUANTITY AS IT RELATES TO UNDERSTANDING THE EFFECTS OF OPERATIONS ON NUMBERS

In the primary grades, students explore the effects of addition and subtraction on quantity: In addition, quantities increase as numbers are added together, and in subtraction, quantities decrease. In the junior grades, students should have opportunities to explore the effects of multiplication and division on number quantities, and they should investigate why:

- multiplying numbers greater than 1 results in a larger quantity (e.g., 4 × 1.5 = 6);
- multiplying a number by a decimal number less than 1 results in a smaller quantity (e.g., 4 × 0.5 = 2);
- dividing a number by a whole number greater than 1 results in a smaller quantity (e.g., 3.5 ÷ 7 = 0.5).

Students should also explore the effects of performing operations using particular numbers, such as multiplying or dividing a number by 10, 100, and 1000, and multiplying a number by 0.1, 0.01, and 0.001.

A solid understanding of fractional quantity will help students develop an understanding of operations with fractions in later grades, and will assist them in avoiding errors that are caused by poor number sense (e.g., adding fractions by adding the numerators and denominators, as in 1/2 + 1/3 and getting an incorrect answer of 2/5 instead of the correct answer of 5/6).

Characteristics of Student Learning

In general, students with a strong understanding of quantity:

- are able to represent quantities using concrete materials and diagrams, and explain how these representations show the "howmuchness" of numbers;
- have mental images of what quantities "look like" (e.g., have a mental image of 3/4);
• consider the size of numbers in various contexts, and judge whether the numbers are reasonable;
• understand the base ten relationships in our number system, and are able to compare whole numbers and decimal numbers by considering the place value of the digits in the numbers;
• relate numbers to benchmark numbers (e.g., recognize that 3/8 is close to 1/2);
• use common fractions, such as 1/4, 1/2, and 3/4, to estimate quantities (e.g., the jar is about 3/4 full of water);
• recognize the relationship between the denominator of a fraction and the size of the fractional parts (e.g., a circle divided into eighths has smaller fractional parts than a circle of equal size that is divided into sixths);
• have strategies for estimating large quantities (e.g., think about what 20 of an item would "look like", and use the smaller quantity as a reference point for estimating a larger quantity);
• are able to explain the effects of operations on quantity (e.g., multiplication of a whole number by a decimal less than 1 results in a quantity that is less than the whole number);
• are able to explain the effects of operations with particular numbers on quantity (e.g., the effect of adding or subtracting 100; the effect of multiplying or dividing a number by a power of 10);
• can explain whether numbers encountered in the media and on the Internet make sense (e.g., recognize the error in a newspaper report that states that 1 out of 25 people, or 25%, cannot read or write).

**Instructional Strategies**

Students benefit from the following instructional strategies:
• using a variety of concrete materials and diagrams to model the "howmuchness" of whole numbers, decimal numbers, fractions, and percents;
• locating numbers on an open number line by considering their proximity to other numbers on the line;
• discussing their mental visualizations of small and large quantities (e.g., “What does a hundredth look like?”);
• discussing the reasonableness of numbers in various situations;
• discussing and representing (e.g., using concrete materials) the effects of operations on quantity;
• estimating quantities and discussing estimation strategies (e.g., visualizing what a small quantity looks like, and then using that quantity as a reference point for estimating a large quantity);
• providing opportunities to associate a large number with an observable quantity of an item (e.g., collecting and observing 10 000 pennies);
• solving numerical problems that are based on meaningful and interesting contexts.
OPERATIONAL SENSE

Overview

Operational sense allows students to make sense of addition, subtraction, multiplication, and division, and to use these operations meaningfully in problem-solving situations. Students who possess a strong understanding of the operations see the relationships among them and develop flexible strategies for computing with numbers.

In the primary grades, students learn about addition and subtraction by using counting strategies and by combining and partitioning numbers. They also begin to understand that groups of equal size can be combined to form a quantity (a fundamental concept in multiplication). The development of operational sense, especially related to multiplication and division, is a focus of instruction in the junior grades. It is important for teachers to provide meaningful contexts, to help students develop an understanding of the operations, and to connect new concepts about the operations to what they already understand.

The following are key points that can be made about operational sense in the junior grades:

- Operational sense depends on an understanding of addition, subtraction, multiplication, and division, the properties of these operations, and the relationships among them.
- Efficiency in using the operations and in performing computations depends on an understanding of part-whole relationships.
- Students demonstrate operational sense when they can work flexibly with a variety of computational strategies, including those of their own devising.
- Solving problems and using models are key instructional components that allow students to develop conceptual and procedural understanding of the operations.

Algorithms – a structured series of procedures that can be used across problems regardless of the numbers – do have an important place in mathematics. . . . Algorithms should not be the primary goal of computational instruction, however. . . . Calculating with number sense means that one should look at the numbers first and then decide on a strategy that is fitting – and efficient.

(Fosnot & Dolk, 2001b, p.102)
UNDERSTANDING THE OPERATIONS, PROPERTIES OF THE OPERATIONS, AND RELATIONSHIPS AMONG OPERATIONS

Instruction that focuses on the meaning of the operations, the properties of the operations, and the relationships among operations helps students to solve problems and develop strategies for computing with numbers.

Understanding the operations

In the primary grades, instruction about operations focuses on developing students’ understanding of addition and subtraction. Students in the primary grades also explore multiplication and division, and are able to solve problems involving these operations by using strategies that make sense to them (e.g., counting out equal groups, using repeated addition, using repeated subtraction). Throughout the junior grades, students’ multiplicative thinking – reasoning that involves ideas such as “three times as many” – continues to develop. Central to the ability to understand multiplication, and eventually division, is the concept of unitizing – the ability to recognize that a group of items can be considered as a single entity (e.g., in $5 \times 8$ or “5 groups of 8”, each group of 8 represents a single entity).

In the past, the focus on developing conceptual understanding of the operations has not been prevalent in many classrooms. Research (Ma, 1999) indicates that many students are taught computational algorithms as a series of steps, with little focus on the understanding of underlying concepts and connections between operations. For example, many students are taught the standard division algorithm without first connecting the operation of division to multiplication and repeated subtraction. Students who rely solely on memorized procedures are often unable to use mathematical reasoning to solve problems. The time spent on promoting conceptual understanding does not hinder and does contribute to the development of computational efficiency (Fuson, 2003).

To develop a deep understanding of the operations, students should have experiences in solving a variety of problems that highlight various meanings of the operations. For example, students should solve division problems that involve both partitive situations and quotative situations. In partitive division (also called “distribution division” or “sharing division”), the whole amount and the number of groups are known, but the number of items in each group is unknown (e.g., “Matthew wants to share 42 marbles equally among his 7 friends. How many marbles will each friend get?”). In quotative division (also called “measurement division”), the whole amount and the number of items in each group are known, but the number of groups is unknown (e.g., “Sara is putting 42 marbles into bags. She put 6 marbles into each bag. How many bags does she need?”). Investigations with a variety of problem types provide students with meaningful contexts in which they can develop and consolidate their understanding of the operations.

When instruction begins with what children know, and the teacher works with their ideas and methods before the introduction of formal rules, then students understand the concepts more deeply. Beginning with student methods and working with these ideas to move toward more effective methods keeps students thinking about the mathematical concepts, trying to make sense of their work and eventually deepens their understanding. The mathematics does not end with the students’ own methods but it should begin with them. (Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004, p. 13)
Properties of the operations

Understanding the properties of the operations allows students to develop flexible and effective mental computation strategies. The properties should be introduced and emphasized in problem-solving situations, and should not be taught in isolation as rules to be memorized. The use of models (e.g., concrete materials, diagrams) helps students to visualize the properties and to recognize their utility as problem-solving strategies. For example, to introduce the distributive property and to demonstrate its use in multiplication, teachers might have students use grid paper to represent tiles arranged in a $6 \times 14$ array. To determine the number of tiles, students could partition the $6 \times 14$ array into a $6 \times 10$ array and a $6 \times 4$ array, calculate the number of tiles in each smaller array, and then add the partial products.

Array Modelling $6 \times 14$

After experiences with representing the distributive property using a grid, students can apply the property in performing mental computations. For example, $5 \times 34$ can be calculated by decomposing 34 into $30 + 4$, then multiplying $5 \times 30$ and $5 \times 4$, and then adding $150 + 20$.

Some of the properties that students should investigate and apply are the following:

- **Identity property**: In addition and subtraction, the identity element is 0, which means that adding 0 to or subtracting 0 from any number does not change the number’s value (e.g., $36 + 0 = 36$; $17 - 0 = 17$). In multiplication and division, the identity element is 1 (e.g., $88 \times 1 = 88$; $56 \div 1 = 56$).

- **Zero property of multiplication**: The product of any number and 0 is 0 (e.g., $98 \times 0 = 0$).

- **Commutative property**: In addition and multiplication, numbers can be added or multiplied in any order, without affecting the sum or product of the operation (e.g., $2 + 99 = 99 + 2$; $25 \times 4 = 4 \times 25$).

- **Associative property**: In addition and multiplication, the numbers being added or multiplied can be regrouped in any way without changing the result of the operations (e.g., $(27 + 96) + 4 = 27 + (96 + 4)$; $(17 \times 25) \times 4 = 17 \times (25 \times 4)$).
Distributive property: A number in a multiplication expression can be decomposed into two or more numbers. The distributive property can involve:

- multiplication over addition (e.g., $6 \times 47 = (6 \times 40) + (6 \times 7)$, which gives $240 + 42 = 282$);
- multiplication over subtraction (e.g., $4 \times 98 = (4 \times 100) - (4 \times 2)$, which gives $400 - 8 = 392$);
- division over addition (e.g., $72 \div 6 = (60 \div 6) + (12 \div 6)$, which gives $10 + 2 = 12$);
- division over subtraction (e.g., $4700 \div 4 = (4800 \div 4) - (100 \div 4)$, which gives $1200 - 25 = 1175$).

Note: It is not necessary for students to know the names of these properties (nor should they be required to memorize definitions of the properties), but it is important that they understand and recognize the usefulness of the properties in performing computations.

Relationships among the operations

Recognizing the relationships among operations allows students to develop a deeper understanding of the operations and helps them to develop flexible computation strategies. As the following example illustrates, students, in their early explorations of division problems, apply strategies and operations that they already understand.

Students are given a quotative division situation in which they are asked to put 84 marbles into bags to sell at a yard sale. They are to put 14 marbles in each bag and they need to determine how many bags they will need.

Using counting: Students might count out 84 counters, arrange them into groups of 14, and then count the number of groups. A counting strategy is commonly used by students who are beginning to explore division.
Using repeated addition: Students might repeatedly add 14 until they reach 84, and then go back to count the number of 14s they added.

\[
\begin{array}{cccccc}
14 & 28 & 42 & 56 & 70 \\
+ 14 & + 14 & + 14 & + 14 & + 14 \\
\hline
28 & 42 & 56 & 70 & 84 \\
\end{array}
\]

I added 6 14s.

Using repeated subtraction: Students might subtract 14 from 84 and then continue to subtract 14 until they reach 0.

\[
\begin{array}{cccccc}
84 & 70 & 56 & 42 & 28 \\
- 14 & - 14 & - 14 & - 14 & - 14 \\
\hline
70 & 56 & 42 & 28 & 14 \\
\end{array}
\]

I subtracted 14 6 times

Using doubling: Students might reason that 2 bags would contain 28 marbles, and that 4 bags would contain 56 marbles. They conclude that 6 bags are needed, since \(28 + 56 = 84\).

Using multiplicative thinking: Students might reason that 10 groups of 14 is 140 (which is greater than 84) but recognize that half of 140 is 70. Since 5 groups of 14 is 70 (which is close to 84), they might add 1 more group of 14 to 70 to get 84 and conclude that 6 bags are needed.
All the strategies described in the preceding example lead to a correct solution; however, some strategies are more efficient than others (i.e., the efficiency and sophistication of the methods increase as students move from simple counting, to additive strategies, and, eventually, to multiplicative strategies). It is important for teachers to remember that students need time, a variety of experiences with the operations, and modelling by teachers and peers before they can construct these relationships for themselves.

**PART-WHOLE RELATIONSHIPS IN USING THE OPERATIONS AND PERFORMING COMPUTATIONS**

In the primary grades, students explore part-whole relationships by composing (putting together) and decomposing (taking apart) numbers (e.g., 3 and 4 make 7; 25 is 20 and 5). These explorations allow students to develop an understanding of number quantity (e.g., the meaning of “7”), place value (e.g., 25 is 2 tens and 5 ones, or 1 ten and 15 ones), addition (combining numerical parts to create a whole), and subtraction (partitioning a whole into parts).

An understanding of part-whole relationships continues to be an important concept when students learn about multiplication and division. In multiplication, groups of equal size are combined to create a whole (e.g., 5 groups of 8 makes 40); and in division, the whole is partitioned into equal groups (e.g., 40 divided into groups of 8 results in 5 groups).

The knowledge of how numbers can be composed and decomposed is also fundamental in performing computations. For example, given a problem involving 47 + 26, students could take apart and combine numbers in different ways to generate a variety of mental computation strategies, including the following:

- increasing 47 to 50 by taking 3 from 26, to get 50 + 23 = 73; or
- adding the tens first to get 40 + 20 = 60; next, adding the ones to get 7 + 6 = 13; finally, adding the subtotals to get 60 + 13 = 73; or
- adding the first addend to the tens of the second addend to get 47 + 20 = 67; then adding the ones from the second addend to get 67 + 6 = 73.

**WORKING FLEXIBLY WITH COMPUTATIONAL STRATEGIES**

Students should develop confidence in their ability to solve problems involving operations and develop efficient strategies for solving such problems. Teachers need to encourage students to use strategies that make sense to them. However, when students’ methods prove to be cumbersome, teachers should guide students in developing increasingly efficient strategies by providing carefully selected problems that lend themselves to learning new strategies. Sharing a variety of student solutions for problems, and modelling various strategies by teachers and students, as well as discussing the efficiency, relevancy, and accuracy of various strategies, will allow students to observe and learn new strategies.
Students should have opportunities to explore and develop a variety of strategies and algorithms before they are introduced to standard algorithms. When students solve problems using their own strategies, they develop contextual, conceptual, and procedural understandings of the operations; and this helps to bring meaning to the steps involved when performing standard algorithms. For example, the illustration below shows different strategies and algorithms for computing $224 \div 17$.

### DEVELOPING UNDERSTANDING OF THE OPERATIONS THROUGH SOLVING PROBLEMS AND USING MODELS

It is important for teachers to engage students in problem-solving experiences that allow them to explore, develop, practice, and consolidate operational concepts and procedures. Students should be encouraged to use both concrete materials and pictorial representations to model problem situations and strategies. Models serve as tools for students to analyse problem situations, to investigate possible strategies, and to conceptualize the role of operations in solving the problems. Usually, the selection of models should be made by students, although, at times, teachers may demonstrate the use of specific concrete materials or written representations (e.g., diagrams, graphic organizers). In the junior grades, number lines, arrays, and open arrays are important models that allow students to represent operations.

#### Number Line Modelling $26 + 45$

Problems should be the starting place for developing arithmetic understanding, thereby establishing the need and the context for computation skills. (Burns, 2000, p. 13)
Teachers should be clear about their purposes for providing problems. For example, problems can be designed or selected with the following purposes in mind:

- conceptual understanding (e.g., to develop students’ understanding of division);
- procedural understanding (e.g., to develop multiplication strategies);
- understanding of relationships (e.g., to help students explore how division is related to multiplication);
- understanding of properties (e.g., to demonstrate how the distributive property can be used to multiply).

Problems can address more than one purpose. For example, students might be asked to determine the number of windows on the side of a building if there are 24 rows of windows with 12 windows in each row. Students might create a $12 \times 24$ array using square tiles to represent the windows, and then determine the total number of windows. The problem-solving experience accomplishes the following purposes:

- conceptual understanding: students connect multiplication to an array;
- understanding of properties: students apply the distributive property by dividing the $12 \times 24$ array into a $10 \times 24$ array and a $2 \times 24$ array, then calculating the number of tiles in the small arrays (240 and 48), and then adding the sub-products (240 + 48) to determine the number of windows.
Characteristics of Student Learning

In general, students with strong operational sense:
• are able to explain the meanings of addition, subtraction, multiplication, and division in various contexts;
• understand the relationships among the operations (e.g., the inverse relationship between multiplication and division);
• apply appropriate operations and strategies in a variety of problem-solving situations;
• develop and apply a range of computational strategies and algorithms, including those of their own devising;
• use strategies and algorithms that make sense to them;
• are able to understand and apply properties of the operations when performing computations (e.g., use the distributive property to compute $8 \times 43$ by multiplying $8 \times 40$ and $8 \times 3$, and then adding the partial products to get $320 + 24 = 344$);
• use a variety of strategies flexibly to perform mental computations;
• are able to demonstrate and explain the operations using models (e.g., base ten blocks, number lines, open arrays);
• determine an appropriate method of computation (e.g., estimation, mental computation, paper-and-pencil calculation, calculator) by considering the context and the numbers involved in the computation;
• use addition, subtraction, multiplication, and division flexibly (e.g., recognize that many division situations can also be solved using multiplication).

Instructional Strategies

Students benefit from the following instructional strategies:
• exploring concepts and procedures related to the operations through problem-solving situations with meaningful contexts;
• exploring various strategies for solving problems, and reflecting on the efficiency and accuracy of different strategies;
• posing and solving their own problems;
• solving a variety of problem types (e.g., solving both quotative and partitive division problems);
• using concrete materials and diagrams (e.g., number lines, open arrays) to model situations, concepts, and procedures related to the operations;
• estimating solutions to problems, and discussing whether estimates are reasonable;
• discussing various estimation strategies, and reflecting on the efficiency and accuracy of different strategies;
• exploring a variety of algorithms, and discussing the meaning of the procedures involved in the algorithms;
• applying and discussing the properties of the operations in problem-solving situations;
• exploring relationships among the operations (e.g., the inverse relationships between addition and subtraction; the inverse relationships between multiplication and division);
• providing opportunities to learn basic multiplication and division facts;
• providing opportunities to develop and practice mental computation strategies.
Overview

To understand mathematics is to recognize the relationships it involves, which give it both its beauty and its utility. Seeing relationships between numbers helps students make powerful connections in mathematics, allowing them to discover new mathematics in flexible, efficient, and innovative ways.

In the junior grades, students’ understanding of base ten relationships and place value expands as they begin to work with larger whole numbers and with decimal numbers. Recognizing number relationships also allows students to understand the operations of addition, subtraction, multiplication, and division, and helps them to develop strategies for computing with numbers in flexible ways.

Mathematical development in the junior grades is also marked by students’ growing knowledge of fractions, percents, and ratios. Understanding these number forms depends on recognizing the relationships that are inherent in numerical expressions.

RELATIONSHIPS IN OUR NUMBER SYSTEM

Having an understanding of relationships in our number system is crucial for making sense of multidigit whole numbers and decimal numbers, and for developing meaningful computational strategies. Students with a strong understanding of our number system comprehend the following concepts:
• Our number system is based on ten-relationships (e.g., 10 ones make 1 ten, 10 tens make 1 hundred, 10 hundreds make 1 thousand).
• Decimal numbers are an extension of the base ten system (e.g., 10 tenths make 1, 10 hundredths make 1 tenth, 10 thousandths make 1 hundredth).
• The position of a digit determines its value (e.g., in 5347, the 3 signifies 3 hundreds).
• Zero in a number indicates the absence of a place-value quantity (e.g., in 6.05, there are no tenths).
• A number can be decomposed according to the place value of its digits (e.g., 485 is 4 hundreds, 8 tens, 5 ones; 27.82 = 20 + 7 + 0.8 + 0.02).
• Number quantities can be regrouped. For example, 417 can be thought of as:
  - 4 hundreds, 1 ten, 7 ones;
  - 3 hundreds, 11 tens, 7 ones;
  - 3 hundreds, 10 tens, 17 ones; and so on.

It is important to note that although materials such as base ten blocks, strips of paper divided into tenths, and $10 \times 10$ grids can help students develop concepts about place value, the concepts are not inherent in the materials. Students must have opportunities to construct their own understanding of place value through interaction with the materials, using the materials as tools for representing ideas and for solving problems.

RELATIONSHIPS IN COMPARING AND ORDERING

Numbers, as representations of quantity, can be compared and ordered. The development of strategies for comparing and ordering numbers helps students to understand the relative size of numbers, and provides them with skills for solving real-life problems. It is important that students have opportunities to compare and order numbers using a variety of manipulatives (e.g., base ten blocks, fraction models). Concrete experiences help students to comprehend the processes involved in comparing and ordering, and allow them to visualize the relative size of numbers. These experiences also help students to develop comparing and ordering strategies that involve looking at written numbers and recognizing the quantities represented by their digits. For example, when comparing whole numbers and decimal numbers, students can consider the place value of the digits within the numbers (e.g., to compare 0.457 and 0.8, students might reason that in 0.457, there are 4 tenths – a value that is less than 8 tenths – and conclude that 0.457 is less than 0.8).

Relating numbers to benchmarks is a useful strategy for comparing and ordering fractions. For example, to order $3/8$, $5/6$, and $7/8$ from least to greatest, students might consider the relative proximity of the fractions to the benchmarks of 0, 1/2, and 1:
• $3/8$ is less than $1/2$ (4/8);
• $5/6$ is $1/6$ less than $6/6$ and therefore is close to 1;
• $7/8$ is $1/8$ less than $8/8$, and is also close to 1.
Students might realize that 3/8 is the only fraction in the set that is less than 1/2 and therefore it is the least. To compare 5/6 and 7/8, students might reason that sixths are larger fractional parts than eightths, and so 5/6 is less than 7/8.

As discussed earlier on pp. 16–17, the open number line provides a model for representing the relative quantities of numbers. As such, it is also an effective tool for comparing and ordering numbers. As the following diagrams indicate, comparing and ordering numbers on an open number line helps students to think about the relative size of the numbers being compared and ordered, and allows students to reflect on the proximity of the numbers to other significant numbers (e.g., whole numbers that are close to decimal numbers or fractions).

Decimal Number Line

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>4.3</th>
<th>4.5</th>
<th>4.9</th>
<th>5</th>
</tr>
</thead>
</table>

Fraction Number Line

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1/8</th>
<th>1/4</th>
<th>3/8</th>
<th>1</th>
</tr>
</thead>
</table>

Related to concepts about number comparisons is the notion of equivalency – that different numerical representations can signify the same quantity (e.g., 1/2 = 2/4 = 5/10; 1.2 = 1.20). An understanding of equivalency also involves knowing how different number types can represent the same quantity (e.g., 4/5 = 0.8 = 80%).

RELATIONSHIPS AMONG THE OPERATIONS AND IN PERFORMING COMPUTATIONS

When students understand how the operations are related, they find flexible ways of computing with numbers. Addition and subtraction are related as inverse operations, as are multiplication and division. Also, multiplication and addition are related (multiplication as repeated addition); and division and subtraction are connected in a similar way (division as repeated subtraction).

Knowing how numbers can be composed (put together) and decomposed (taken apart) is central to understanding the operations (addition, subtraction, multiplication, division) and to developing flexible computational strategies. In the primary grades, students learn that numbers can be decomposed in different ways (e.g., 8 can be broken down into 1 and 7, or 2 and 6, or 3 and 5, and so on). Understanding that a whole can be broken into parts allows students to develop strategies for addition and subtraction. For example, if students are unsure of the sum of 6 + 8, they might add 6 + 4 to get 10 and then add the remaining 4 from 8 to get 10 + 4 = 14. In the junior grades, students continue to apply composing and decomposing strategies to add and subtract larger numbers. To subtract 54 – 27, for example, students could decompose 27 into 20 and 7, subtract 20 from 54 (to get 54 – 20 = 34), and then subtract 7 from 34 (to get 34 – 7 = 27).

Composition and decomposition of number also play a significant role in students’ understanding of multiplication and division. When students are given opportunities to represent...
multiplication (composing quantities by combining groups of equal size) and represent division (decomposing quantities by partitioning into equal groups), they observe number relationships in basic multiplication and division facts (e.g., $48 = 6 \times 8$ and $48 \div 8 = 6$).

Number representations that use arrays also allow students to recognize how numbers are related in multiplication. The following diagram illustrates how $6 \times 14$ can be decomposed into $6 \times 5$ and $6 \times 9$. When students learn to decompose multiplication expressions (i.e., they understand the distributive property), they gain a powerful tool for computing with numbers mentally.

![Diagram illustrating multiplication as decomposing quantities](image)

Development of concepts related to the operations influences the kinds of reasoning that students can bring to mathematical situations. Beyond seeing additive relationships (relationships involving addition and subtraction, such as 2 more or 3 less), students begin to recognize multiplicative relationships (relationships involving multiplication and division, such as 10 times more, 3 times smaller, half the size). Understanding multiplicative relationships allows students to develop skills in proportional reasoning.

**RELATIONSHIPS AMONG FRACTIONS, DECIMAL NUMBERS, AND PERCENTS**

In the junior grades, students learn that fractions, decimal numbers, and percents are related to one another. They recognize decimal numbers as base ten fractional representations that make use of place value (i.e., decimal numbers represent fractions with denominators of 10, 100, 1000, and so on).

![Fraction Strip (in Tenths) Showing How Much Is Shaded](image)
Students also learn that percents are fractions that are based on a one-hundred-part whole. Models, such as $10 \times 10$ grids, allow students to recognize the relationship among fractions, decimal numbers, and percents.

**$10 \times 10$ Grid Showing How Much Is Shaded**

As students establish these relationships, they are able to identify common decimal and percent equivalents for various fractions (e.g., $1/4$ is equivalent to $25/100$ and can be written as $0.25$ or $25\%$), and select the representation that is most useful in a given situation. Consider the following problem.

*"In a bag of 24 marbles, 25% of the marbles were red. How many red marbles were in the bag?"

To solve this problem, a student who is familiar with the relationship between percents and fractions might reason that $25\%$ is the same as $1/4$, and then determine that $1/4$ of $24$ is $6$.

In the junior grades, the focus should be on the use of models to show the relationship among fractions, decimal numbers, and percents, rather than on learning rules for converting among forms.

**Characteristics of Student Learning**

In general, students with a strong understanding of relationships:

- understand base ten relationships in our number system (e.g., a $3$ in the hundreds place is $10$ times the value of a $3$ in the tens place, and $100$ times the value of a $3$ in the ones place);
- use their understanding of base ten relationships in our number system to think about number size, compare numbers, and perform computations;
- apply effective strategies for comparing and ordering numbers (e.g., consider place value in whole numbers and decimal numbers);
• relate fractions to the benchmarks of 0, 1/2, and 1;
• identify and describe the relationships that exist among fractions, decimal numbers, and percents;
• use their understanding of number relationships when performing computations (e.g., to compute 37 + 49, students might add 37 + 50 first to get a sum of 87, and then take 1 from 87 to account for the difference between 49 and 50);
• compose and decompose numbers flexibly (e.g., to multiply 4 × 28, students might decompose 28 into 20 + 8, multiply 4 × 20 and 4 × 8, and then add the partial products to get 80 + 32 = 112);
• develop a sense of the relationships among operations (e.g., use the inverse relationship between multiplication and division to solve problems involving either operation);
• understand additive relationships (e.g., 10 more than, 25 less than), and begin to recognize multiplicative relationships (e.g., 10 times more, 3 times smaller, half the size).

**Instructional Strategies**

Students benefit from the following instructional strategies:

• representing whole numbers and decimal numbers using a variety of materials (e.g., base ten blocks, place-value charts, fraction strips divided into tenths, 10 × 10 grids), and discussing base ten relationships in numbers;
• using number lines, arrays, and place-value charts to develop an understanding of relationships between numbers;
• decomposing numbers in various ways (e.g., 1367 is 1 thousand, 3 hundreds, 6 tens, 7 ones, or 13 hundreds, 6 tens, 7 ones; 475 = 400 + 75, or 500 – 25);
• comparing and ordering whole numbers, decimal numbers, and fractions using a variety of methods (e.g., using concrete materials, drawing number lines);
• representing fractions using concrete materials and drawings, and discussing the proximity of fractions to the benchmarks of 0, 1/2, and 1;
• solving problems using a variety of strategies, and discussing the relationships between strategies (e.g., discussing how a problem might be solved using skip counting, repeated addition, and multiplication);
• providing opportunities to perform mental computations, and discussing various strategies used.
Overview

Representation is an essential element in supporting students’ understanding of mathematical concepts and relationships. It is needed when communicating mathematical understandings and recognizing connections among related mathematical concepts. Representations are used in applying mathematics to problem situations through modelling (National Council of Teachers of Mathematics [NCTM], 2000).

In the primary grades, students represent numbers up to 1000, use notation involving the four operations (addition, subtraction, multiplication, division), and are introduced to the symbols for fractions. In the junior grades, this knowledge is extended to include larger numbers up to 1,000,000 and new symbols (e.g., decimal point, percent sign, ratio). Students understand that making written representations of mathematical ideas is an essential part of learning and doing mathematics. They are encouraged to represent ideas in ways that make sense to them, even if these representations are not conventional ones. However, students also learn conventional forms of representation to help with both their own learning of mathematics and their ability to communicate with others about mathematical ideas.

The following are key points that can be made about representation in the junior grades:
- Symbols and placement are used to indicate quantity and relationships.
- Mathematical symbols and language, used in different ways, communicate mathematical ideas in various contexts and for various purposes.

Using Symbols and Placement to Indicate Quantity and Relationships

Numerical symbols, and their position within a number or notation, are used to indicate quantity and relationships.

There is no doubt that one of the most significant mathematical concepts that students at all levels need to understand is that of place value and the role that place value plays in the way we represent numbers. It is this convention that allows us to infinitely extend our number
system to include very large numbers and very small numbers. The positions of digits in whole numbers and decimal numbers determine what the digits represent (i.e., the size of group they count). For example, the 4 in 40 represents 4 tens; the 4 in 4 000 000 represents 4 millions; the 4 in 0.004 represents 4 thousandths, and so on. In particular, students in the junior grades need to recognize the decimal point as a convention that "announces" the separation between the whole-number part of a quantity and the part that is less than one whole.

The symbolic representation of a fraction is a convention that extends our number system to infinitely smaller parts. The bottom number (denominator) denotes the size of the part and describes the number of parts in the whole. The top number (numerator) counts the number of parts and tells how many are selected. The representation for fractions, however, can be very confusing for students. Fractions use whole numbers in the notation, and nothing in that notation or the words used to describe it conveys their meaning as "parts". So, for example, the fraction \( \frac{3}{8} \) is read as "three eighths", which doesn’t really convey the intended meaning that there are 8 equal parts and of these 8 parts, 3 are selected. To help students create meaning from fractional notations, teachers should provide them with many experiences that involve partitioning quantities into equal parts using concrete models, pictures, and meaningful contexts. Introducing the standard notation for fractions needs to be done in such a way as to ensure that students can connect the meanings already developed for these numbers with the symbols that represent them.

Although understanding that a fraction describes a relationship to a whole or a part of a group has already been discussed, it is also important to understand that a fraction is a quantity. A fraction can be thought of as a single entity, with its own unique place on the number line – just as 3 is a single entity. As such, fractions, like whole numbers, can be compared and ordered (and operated on).

Although obviously related to fractions, the concept of ratio has its own representations. The conventional symbol for a ratio (\( : \)) describes two quantities in relation to each other. So, although the ratio of "three to four" (3:4) can also be represented symbolically using a fraction (3/4), a decimal number (0.75), or words (3 out of 4), not all these representations are helpful or effective for interpreting what the ratio means. A ratio can express a comparison of a part to a whole, such as the ratio of boys to all the students in the class (e.g., there are 25 students and 15 boys, so the ratio of boys to students is 15:25). It can also
express a comparison of one part of a whole to another part of the same whole, such as the ratio of boys in the class to girls in the class (e.g., there are 15 boys and 10 girls, so the ratio is 15:10).

Recognizing equivalent ratios (e.g., 3 out of 4 marbles being blue represents the same ratio as 9 out of 12 marbles being blue) is fundamental for developing proportional reasoning.

**USING SYMBOLS AND LANGUAGE IN DIFFERENT WAYS TO COMMUNICATE MATHEMATICAL IDEAS**

In the junior grades, students become more aware that a symbol can have different meanings, depending on the representation used (e.g., the meaning of “1” is different in 1, 100, 0.1, 1%, and so on).

What is not as commonly addressed, but is of significance in developing a deeper conceptual understanding of number, is that different representations can be used to more clearly communicate and emphasize particular relationships. Consider the following different representations for the number 144:

\[ 12^2 \quad 144 \quad 2 \times 2 \times 2 \times 2 \times 3 \times 3 \quad 9 \times 16 \]

Teachers might choose one of these representations over the others to emphasize a certain characteristic of the number. For example, if teachers want to show that 144 is a perfect square, the first representation helps to make that clear. If teachers want to show that 144 is divisible by 9, the last representation might be selected. Students in the junior grades become aware not only that numbers may have different representations but that these representations might be used to illustrate or explain different characteristics/properties of the number.

**Triple Number Lines Showing Different Number Forms for the Same Quantity**

<table>
<thead>
<tr>
<th>0%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/4</td>
<td>1/2</td>
<td>3/4</td>
<td>1</td>
</tr>
<tr>
<td>0.00</td>
<td>0.25</td>
<td>0.50</td>
<td>0.75</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Double Number Lines Showing Proper Fractions, Improper Fractions, Whole Numbers, and Mixed Numbers**

<table>
<thead>
<tr>
<th>1/2</th>
<th>1/4</th>
<th>3/4</th>
<th>1</th>
<th>1 1/2</th>
<th>1 1/3</th>
<th>1 1/4</th>
<th>1 1/2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/2</td>
<td>1/4</td>
<td>1/2</td>
<td>1</td>
<td>1 1/2</td>
<td>1 1/3</td>
<td>1 1/4</td>
<td>2</td>
</tr>
</tbody>
</table>

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**Number Sense and Numeration, Grades 4 to 6 – Volume 1**
Students at this level realize that not only mathematical symbols but also mathematical words can assume new meanings and representations. For example, in the primary grades students learn that “sixth” means a “position” (I am the sixth person in the line) and it is an ordinal number; but, in the junior grades, that same word when used in the context of fractions might mean “1 of 6 equal parts or groups” (a sixth of the pie was eaten).

As students access mathematical representations and the ideas they represent, they have a set of tools that significantly expand their capacity to think mathematically.

**Characteristics of Student Learning**

In general, students with a strong understanding of representation:

- are able to represent quantities using concrete materials and diagrams;
- recognize that a fraction is a representation whose denominator tells how many parts a whole is divided into, and whose numerator tells how many parts there are;
- know how to represent very large numbers but may be confused about how to read them. For example, 1,100,000 is one million, one hundred thousand. A common error is to forget that and means “decimal point” and say, “one million and one hundred thousand”;
- understand the base ten relationships in our number system, and are able to recognize, for example, that 0.255 is less than 0.789;
- read large numbers correctly, such as 2,200,000, but become confused if the number is represented as 2.2 million;
- understand the representations of common fractions and recognize that the size of the whole is important to the representation (e.g., 1/8 of a large pizza is more than 1/8 of a small one);
- can link one representation of an amount to another representation (e.g., 1100 is the same as 11 hundreds, or 110 tens, or 1100 ones);
- can move flexibly among five different representations of mathematical ideas: pictures, written symbols, oral language, real-world situations, and manipulative models.

**Instructional Strategies**

Students benefit from the following instructional strategies:

- using concrete and pictorial representations and linking them to mathematical words, terminology, and symbolic notation (e.g., using double and triple number lines that show different representations of the same idea);
- providing opportunities to recognize that different number forms can represent the same quantity (e.g., 2/5 = 0.4 = 40%);
- examining many mathematical representations over time and in real-life situations (e.g., reading newspaper headlines and discussing what $1.2 million would look like if written with just words or numbers).
• providing opportunities to use self-initiated and teacher-suggested drawings of mathematical concepts and procedures (e.g., using a closed array and an open array to develop concepts and procedures related to multiplication and division);
• providing opportunities to connect standard notation for fractions (i.e., proper fractions, improper fractions, and mixed numbers) to the meanings already developed for these numbers and the symbols that represent them;
• providing many experiences to use a variety of tools (e.g., number lines, calculators, hundreds charts), to help students develop an understanding of how the movement of a digit to the right or left significantly alters its value, and to help students read the new numbers that each movement makes;
• providing experiences for students to orally say the names of symbols and to read numbers (e.g., saying “and” to distinguish between wholes and parts in reference to a decimal number; 2 756.032 is two thousand seven hundred fifty-six and thirty-two thousandths);
• providing opportunities to represent mathematical ideas in a variety of ways that make sense to them, even if these representations are not conventional ones.
PROPORTIONAL REASONING

Overview

The ability to compare an object or a set of objects to another helps students make sense of, organize, and describe their world. Young children begin by making qualitative comparisons (e.g., bigger, smaller). Through experience, students learn to make additive comparisons (e.g., 2 more than, 50 less than) and eventually progress to making multiplicative comparisons (e.g., three times as long as, half as much). Consider the following example.

Qualitative comparison: The adult is taller than the child.
Additive comparison: The adult is 100 cm taller than the child.
Multiplicative comparison: The adult is twice as tall as the child.

Proportional reasoning is the capstone of the elementary curriculum and the cornerstone of algebra and beyond.

(Post, Behr, & Lesh, 1988)
The ability to think about the multiplicative relationship between two ratios is fundamental to proportional reasoning. In the preceding example, knowing that the adult is twice as tall as the child involves recognizing that the ratio of 200 to 100 is equal to the ratio of 2 to 1 (the second value in each ratio is twice the first value).

Proportional reasoning is a significant development in students’ mathematical thinking. It represents the ability to consider the relationship between two relationships – to understand not only the relationship between values in a ratio but also the relationship between two or more ratios. The following example illustrates the multiplicative relationship between two ratios.

"Four girls share 12 mini-pizzas equally, and 3 boys share 9 mini-pizzas equally. Who gets to eat more pizza?"

The ratio of the number of girls to the number of pizzas is 4:12.

The ratio of the number of boys to the number of pizzas is 3:9.

In both cases, the number of mini-pizzas is three times the number of children.

Since the ratios are equal, both girls and boys get the same amount of pizza.

It is important that students in the junior grades be provided with experiences that help them begin to reason proportionally. These informal experiences provide the background for the more formal study, in later grades, of several topics including proportions, percent, similarity, and algebra.

PROPORTIONAL REASONING AS MULTIPLICATIVE COMPARISONS BETWEEN RATIOS

Proportions involve two or more equivalent ratios. For example, if orange juice is made by mixing 1 part juice concentrate to 3 parts water (a ratio of 1:3), then orange juice made with 3 parts concentrate needs to be mixed with 9 parts water (a ratio of 3:9). The concentrate-water ratios, 1:3 and 3:9, are equivalent – the concentrate and water parts in the first ratio (1 and 3) are both multiplied by 3 to generate the second, equivalent ratio.

Initially, students in the junior grades may make additive comparisons when trying to make sense of proportional situations. For example, students who are beginning to explore proportionality might conclude that 3 parts of concentrate need to be mixed with 5 parts of water (thinking that a concentrate-to-water ratio of 1:3 is equal to a ratio of 3:5 because the difference between the numbers in each ratio is 2). It is important that students have opportunities to represent proportional relationships using concrete materials and diagrams, so that they can observe the multiplicative relationships inherent in proportions.
Ratio tables can be very helpful in enabling students to identify and extend proportional relationships. For more on ratio tables, see the glossary.

<table>
<thead>
<tr>
<th>Orange juice concentrate</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>3</td>
<td>9</td>
<td>?</td>
<td>?</td>
<td>6</td>
</tr>
</tbody>
</table>

**EXPRESSING PROPORTIONAL RELATIONSHIPS USING FRACTIONS, RATIOS, AND PERCENTS**

Proportional relationships can be expressed using fractions, ratios, and percents: for example, about 1/3 of all homes have a pet dog; the ratio of the team’s wins to losses is 2 to 3; sale items are 10% off the regular price. Students learn that the fractions, ratios, and percents used to express proportional relationships do not represent discrete quantities, but that they infer ideas such as “out of every”, “for every”, “compared with”, and “per”. In the preceding examples, the numbers do not refer to exactly 3 homes, or 2 and 3 games, or $10; but rather, 1 house out of every 3; 2 wins for every 3 losses; $10 per $100. With an understanding of such relationships, students can reason proportionally to determine exact quantities.

In the examples, if about 1/3 of all homes have a pet dog, then about 20 out of 60 homes have a dog; if the ratio of a team’s wins to losses is 2 to 3 and 15 games were played, the team won 6 games and lost 9; if items are $10 off, the discount on an item that regularly sells for $25 is $2.50.

**DEVELOPING PROPORTIONAL REASONING THROUGH INFORMAL ACTIVITIES**

Students in the junior grades should have opportunities to solve a variety of problems that involve proportional reasoning. Proportional-reasoning problems can involve:

- determining a missing value in a ratio. For example, Amy and Habib were biking at the same speed. It took Amy 1.5 hours to bike 24 km. How long did it take Habib to bike 20 km?
• comparing two ratios. For example, Amy biked 12 km in 45 minutes, and Habib biked 8 km in 40 minutes. Who biked at a faster speed?
• making qualitative comparisons. For example, Amy and Habib biked the same number of kilometres. It took Amy 35 minutes and it took Habib 30 minutes. Who biked faster?
• determining unit rates. For example, Amy biked 18 km in 1.5 hours. What was her speed in kilometres per hour?
• making conversions involving measurement units. For example, It took Habib 15 minutes to bike 3550 m. What was his speed in kilometres per hour?
• determining a fraction of a quantity. For example, Amy wants to bike a distance of 24 km. So far, she has biked 16 km. What fraction of the total distance has she biked?

Teaching mechanical procedures for determining equal ratios (e.g., using cross-multiplication, solving for the unknown variable) does little to promote the development of proportional reasoning. Conceptual understanding about proportional situations emerges when students apply strategies that they find personally meaningful. As illustrated below, students initially use concrete materials and diagrams to help them reason about proportional situations.

"A CD store allows you to trade in 3 of your CDs to get a different used CD. How many CDs could you get if you traded in 12 CDs?"

Students also make connections between proportional reasoning and mathematical processes that they already understand. For example, students recognize that determining equivalent ratios is similar to determining equivalent fractions. A ratio of 3:5 is equivalent to a ratio of 6:10, just as 3/5 = 6/10.

Note: A distinction between equivalent ratios and equivalent fractions must be made. Equivalent fractions represent the same quantity (e.g., 3/5 of a cord is the same length as 6/10 of a cord). Equivalent ratios, however, can represent different quantities (e.g., a bowl of 3 apples and 5 plums is a different quantity than a bowl of 6 apples and 10 plums; however, the ratios of apples to plums in both bowls are equivalent).

Students, with guidance from teachers, can devise methods to organize proportional thinking. For example, a ratio table is an effective tool for recording equivalent ratios.
“In a large aquarium, there is 1 goldfish for every 3 guppies. How many guppies are there if there are 12 goldfish?”

If there are 12 goldfish, there are 36 guppies.

<table>
<thead>
<tr>
<th>Goldfish</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guppies</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>18</td>
<td>36</td>
</tr>
</tbody>
</table>

Characteristics of Student Learning

In general, students who are developing proportional reasoning:

- distinguish between additive relationships (e.g., 10 more than, 2 less than) and multiplicative relationships (e.g., 10 times more, half as much);
- describe multiplicative relationships between quantities (e.g., “You have three times as many stickers as I have”);
- are able to solve problems involving proportional reasoning (e.g., determine the cost of 12 books if the cost of books is 3 for $5);
- solve problems involving unit rates;
- are able to make qualitative comparisons based on given ratios (e.g., a mixture of 3 parts white paint to 1 part black paint will be lighter than a mixture of 2 parts white paint to 1 part black paint);
- can compare ratios (e.g., a class with a girl-to-boy ratio of 2:3 has proportionally more girls than a class with a girl-to-boy ratio of 2:4);
- represent proportional relationships using concrete materials, drawings, and ratio tables;
- express and explain proportional relationships using fractions, ratios, and percents.

Instructional Strategies

Students benefit from the following instructional strategies:

- discussing everyday situations that involve proportional reasoning (e.g., determining the cost of multiple items using unit rates);
- using concrete materials and diagrams to model situations, concepts, and procedures related to proportional reasoning;
- providing opportunities to use and discuss students’ informal strategies (e.g., using concrete materials, drawing diagrams, using ratio tables) for solving problems that involve proportional reasoning;
- exploring and discussing both proportional and non-proportional situations;
- solving a variety of different problem types (e.g., determining a missing value in a ratio, comparing two ratios, determining unit rates, making conversions involving measurement units);
- posing and solving their own proportional-reasoning problems.
REFERENCES


algorithm. A systematic procedure for carrying out a computation. An algorithm that has come into common usage over time is often called a standard algorithm (or traditional algorithm). Traditional algorithms focus on the digits (e.g., “Add the ones, then regroup ones to tens; add the tens, then regroup tens to hundreds; and so on”). An algorithm or strategy that focuses on the numbers involved in the computation is often called a flexible algorithm (strategy) or a non-standard algorithm. The “standard algorithm” for multiplication begins by multiplying 6 times 7 to get 42, and recording the “regrouped” 4 tens above the 5; then multiplying $6 \times 5$ to get 30, and adding the 4 to get 34.

In applying a “flexible algorithm or strategy” for multiplying $6 \times 57$, a student might think of 57 as 50 and 7, and write:

Algorithm

array. A rectangular arrangement of objects into rows and columns, often used to represent multiplication. For example, $3 \times 6$ can be represented by an array showing 3 rows with 6 objects in each row. See also open array.

Example:

\[
\begin{array}{c}
57 \\
\times 6 \\
\hline
342
\end{array}
\]

42, and recording the “regrouped” 4 tens above the 5; then multiplying $6 \times 5$ to get 30, and adding the 4 to get 34.

array.

associative property. A property of addition and multiplication that allows the numbers being added or multiplied to be grouped differently without changing the result. For example, $(77 + 99) + 1 = 77 + (99 + 1)$, and $(7 \times 4) \times 5 = 7 \times (4 \times 5)$. Using the associative property can simplify computation. The property does not generally hold for subtraction or division.

base ten fraction. A fraction whose denominator is a power of 10 (e.g., 3/10, 29/100, 7/1000). Also called a decimal fraction.

base ten materials. Learning tools that can help students learn many number sense concepts (e.g., place value, operations with whole numbers, fractions, and decimals). Sets of base ten materials typically include small cubes called “units” that represent ones; “rods” or “longs” that represent tens (ten “units”), “flats” that represent hundreds (ten “rods” or “longs”), and large cubes that represent thousands (ten “flats”).

benchmark. A number that is internalized and used as a reference to help judge other numbers. For example, 0, 1/2, and 1 are useful benchmarks when comparing and ordering fractions. Also called a referent.

big ideas. In mathematics, the important concepts or major underlying principles. For example, in this document, the big ideas that have been identified for Grades 4–6 in the Number Sense and Numeration strand of the Ontario curriculum are quantity, operational sense, relationships, representation, and proportional reasoning.
commutative property. A property of addition and multiplication that allows the numbers to be added or multiplied in any order without affecting the sum or product. For example, $3 + 99 = 99 + 3$, and $23 \times 2 = 2 \times 23$. Using the commutative property can simplify computation. The property does not generally hold for subtraction or division.

compensation. A mental mathematics strategy in which part of the value of one number is given to another number to make computation easier. For example, $26 + 99$ can be thought of as $25 + 100$; that is, 1 from the 26 is transferred to the 99 to make 100. Compensation sometimes takes place at the end of the computation. For example, $26 + 99$ can be thought of as $26 + 100 = 126$; and since 1 too many was added, take one away to get 125.

composite number. A number that has factors besides itself and 1. For example, 12 is a composite number – it has factors of 1, 2, 3, 4, 6, and 12. See also prime number.

composition of numbers. The putting together of numbers. For example, 4 thousands and 2 hundreds can be composed to make 4200. See also decomposition of numbers and recomposition of numbers.

decomposition of numbers. The taking apart of numbers. For example, the number 402 is usually taken apart as 400 and 2, but it can also be taken apart in other ways, such as 390 and 12. Students who can decompose numbers in many different ways develop computational fluency and have many strategies available for solving arithmetic questions mentally. See also composition of numbers and recomposition of numbers.

denominator. In fractions, the number written below the line. It represents the number of equal parts into which a whole or set is divided, or the divisor of a division sentence. For example, in $\frac{3}{4}$, the denominator is 4 and might mean 4 equal parts, 4 objects in a group, or 3 divided by 4. See also numerator.

distributive properties. The properties that allow numbers in a multiplication or division expression to be decomposed into two or more numbers. These properties include:
• **Distributive property of multiplication over addition**, for example,
  \[ 6 \times 47 = (6 \times 40) + (6 \times 7) \], which results in \(240 + 42 = 282\);

• **Distributive property of multiplication over subtraction**, for example,
  \[ 4 \times 98 = (4 \times 100) - (4 \times 2) \], which results in \(400 - 8 = 392\);

• **Distributive property of division over addition**, for example,
  \[ 72 ÷ 6 = (60 ÷ 6) + (12 ÷ 6) \], which results in \(10 + 2 = 12\);

• **Distributive property of division over subtraction**, for example,
  \[ 4700 ÷ 4 = (4800 ÷ 4) - (100 ÷ 4) \], which results in \(1200 - 25 = 1175\).

**Division**. The operation characterized by the equal sharing of a quantity into a known number of groups (partitive division), or by the repeated subtraction of an equal number of items from the whole quantity (quotative division). Division is the inverse operation of multiplication. The quantity to be divided is called the **dividend**. The number to be divided by is the **divisor**. The **quotient** is the result of a division problem. The **remainder** is the number “left over”, which cannot be grouped or shared equally. For example:

\[
\begin{align*}
\text{dividend} & \quad \text{divisor} \\
67 & \quad 6 \\
\text{quotient} & \quad \text{remainder} \\
11 & \quad 1
\end{align*}
\]

**Equivalent fractions**. Different representations in fractional notation of the same part of a whole or group. For example, 1/3, 2/6, 3/9, and 4/12 are equivalent fractions.

**Estimation strategies**. Mental mathematics strategies used to obtain an approximate answer. Students estimate when an exact answer is not required, and when they are checking the reasonableness of their mathematics work. Some examples of estimation strategies can be found in the introductory pages of Volume 3: Multiplication and Volume 4: Division.

**Factors**. Natural numbers that divide evenly into a given natural number. For example, the factors of 12 are 1, 2, 3, 4, 6, and 12, because all these numbers divide evenly into 12. See also multiplication.

**Friendly numbers**. Numbers that are easy to work with. For example, to calculate 192 ÷ 6, students might think of 192 as 180 and 12, and calculate 180 ÷ 6 and 12 ÷ 6. Also called **compatible numbers**.

**Identity property (identity element)**. The identity property occurs when a number is combined with a special number (identity element) by using one of the operations, and the result leaves the original number unchanged. In addition and subtraction, the special number or identity element is 0; for example, 36 + 0 = 36 and 17 - 0 = 17.

In multiplication and division, the identity element is 1; for example, 88 x 1 = 88 and 56 ÷ 1 = 56.

**Improper fraction**. A fraction whose numerator is greater than its denominator and whose value is greater than 1; for example, 7/3.

**Inverse operations**. The opposite effects of addition and subtraction, and of multiplication and division. Addition involves joining sets; subtraction involves separating a quantity into sets. Multiplication refers to joining sets of equal amounts; division is the separation of an amount into equal sets.
manipulatives. Objects that students handle and use in constructing their own understanding of mathematical concepts and skills, and in demonstrating that understanding. Some examples are base ten materials, fraction circles, and tiles. Also called concrete materials.

mathematical conventions. Agreed-upon rules or symbols that make the communication of mathematical ideas easier.

mental math. Ways of computing mentally, with or without the support of paper and pencil. For example, to calculate 501 – 199 mentally, one might add 1 to both numbers and subtract 502 – 200 = 302.

mixed number. A number greater than 1 that is composed of a whole number and a fraction; for example, 8 1/4.

model. A concrete or pictorial representation of a mathematical idea, which students handle or use to construct their own understanding of mathematical concepts and skills, and to illustrate that understanding; for example, number line, array.

multiple. The product of a given whole number multiplied by any other whole number. For example, 7, 14, 21, 28, 35, ... are multiples of 7.

multiplication. An operation characterized by the combination of equal groups, by repeated addition, or by an array. Multiplication is the inverse operation of division. The multiplication of factors gives a product. For example:

\[ 4 \times 5 = 20 \]

factor \hspace{1cm} factor \hspace{1cm} product

natural number. Any one of the counting numbers 1, 2, 3, 4, ...

number line. A visual model that matches a set of numbers and a set of points one to one. For example:

\[ \begin{array}{c}
0 \hspace{1cm} \frac{1}{4} \hspace{1cm} \frac{1}{2} \hspace{1cm} \frac{3}{4} \hspace{1cm} 1 \\
\end{array} \]

• An open number line consists of a marked but unlabelled number line that can be used to represent various values depending on the starting point and interval selected.

• A double number line is used to represent the equivalencies between two quantities. For example, a double number line that shows decimal numbers and their equivalent fractions might display the decimals sequentially along the top of the number line and display the corresponding equivalent fractions along the bottom of the number line.

number sense. The ability to interpret numbers and use them correctly and confidently.

numerator. In fractions, the number written above the line. It might represent the number of equal parts (or objects of a group) being considered or the dividend of a division sentence. For example, in 3/4, the numerator is 3 and might mean 3 of 4 equal parts, 3 of 4 objects in a group, or 3 divided by 4. See also denominator.
open array. A rectangular arrangement, used to represent multiplication or division, in which the rows and columns of individual objects are not represented. However, the factors of the multiplication expression (the number of implied rows and columns) are recorded on the length and width of the rectangle. An open array does not have to be drawn to scale. For example, $3 \times 67$ might be represented by an open array such as the following:

\[
\begin{array}{c|c}
60 & 7 \\
3 & 3 \times 60 = 180 \\
3 \times 7 = 21
\end{array}
\]

operational sense. An understanding of the mathematical concepts and procedures involved in operations on numbers (addition, subtraction, multiplication, and division) and of the application of operations to solve problems.

partitive division. In partitive division, the whole amount is known and the number of groups is known, while the number of items in each group is unknown. For example, "Daria has 42 bite-sized granola snacks to share among her 6 friends. How many does each friend get?" Also called distribution division or sharing division.

part-part-whole. The idea that a number can be made of two or more parts. For example, 57 can be separated into 50 and 7, or 30 and 20 and 7.

percent. A fraction or ratio, expressed by using the percent symbol, %, in which the denominator is 100. Percent means “out of 100”. For example, 30% means 30 out of 100.

A percent can be represented by a fraction with a denominator of 100; for example, $\frac{30}{100}$.

prime number. A whole number greater than 1 that only has two factors, itself and 1. For example, 13 is a prime number — its only factors are 13 and 1. See also composite number.

proper fraction. A fraction whose numerator is less than its denominator and whose value is less than 1; for example, $\frac{2}{3}$.

proportion. A statement or equation that two or more ratios are equal. For example, $\frac{2}{3} = \frac{6}{9}$; and $1:7 = 2:14$.

proportional reasoning. Reasoning that involves an understanding of the multiplicative relationship in the size of one quantity compared with another. Students express proportional reasoning informally by using phrases such as “twice as big as” and “a third the size of”.

quantity. The “howmuchness” of a number. An understanding of quantity helps students estimate and reason with numbers and is an important prerequisite to understanding place value, the operations, and fractions.

quotative division. In quotative division, the whole amount is known and the number of items in each group is known, while the number of groups is unknown. For example, “Thomas is packaging 72 ears of corn into bags. If each bag contains 6 ears of corn, how many bags will Thomas need?” Also called measurement division.
rate. A comparison, or type of ratio, of two quantities with different units, such as distance and time; for example, a speed of 100 km/h, a gasoline consumption of 20 L/100 km.

ratio. A comparison of quantities with the same units. A ratio can be expressed in ratio form, fractional form, or words; for example, 3:4 or 3/4 or 3 to 4.

rational number. A number that can be expressed as a fraction in which the denominator is not 0.

ratio table. A model that can be used to develop an understanding of multiplication, equivalent fractions, division, and proportional reasoning. For example, the ratio table below would help students solve the following problem: “A grocer sells flour by the kilogram. If a 4 kg bag costs $6.60, how much would a 2 kg, a 3 kg, a 5 kg, and a 6 kg bag cost?” If students know that 4 kg is $6.60, they can easily halve that amount to find the 2 kg amount and halve it again for 1 kg. Students can then calculate 6 kg by adding the 2 kg and 4 kg amounts. 3 kg can be calculated by either halving the 6 kg amount or adding the 2 kg and 1 kg amounts (each student makes a choice based on what is fastest and easiest for him or her). 5 kg can be calculated by either adding the 4 kg and 1 kg amounts or calculating the 10 kg amount, which is easy, and then halving. The table would also help students extend the problem to other amounts.

recomposition of numbers. The putting back together of numbers that have been decomposed. For example, to calculate 298 + 303, a student might decompose the numbers as 298 + 300 + 3, and then recompose the numbers as 300 + 300 + 1 to get the answer 601. See also composition of numbers and decomposition of numbers.

relationship. In mathematics, a connection between mathematical concepts, or between a mathematical concept and an idea in another subject or in real life. As students connect ideas they already understand with new experiences and ideas, their understanding of mathematical relationships develops.

representation. The use of manipulatives, diagrams, pictures, or symbols to model a mathematical concept or real-world context or situation.

rounding. A process of replacing a number by an approximate value of that number to assist in estimation. Note that rounding does not refer to any set of rules or procedures (e.g., look to the number on the right – is it less than 5?). To estimate $5 \times 27$, for example, 27 might be rounded to 30 (to give an estimate of $5 \times 30 = 150$), but 27 could also be rounded to 25 (to give an estimate of $5 \times 25 = 125$).

### Ratio Table

<table>
<thead>
<tr>
<th>Size of bag</th>
<th>4 kg</th>
<th>2 kg</th>
<th>1 kg</th>
<th>6 kg</th>
<th>3 kg</th>
<th>5 kg</th>
<th>10 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$6.60</td>
<td>$3.30</td>
<td>$1.65</td>
<td>$9.90</td>
<td>$4.95</td>
<td>$8.25</td>
<td>$16.50</td>
</tr>
</tbody>
</table>
simplification. In division, the process of “reducing” or multiplying a given problem to make a friendlier problem. For example, 128 ÷ 32 has the same quotient as 64 ÷ 16 (halve both numbers), or 32 ÷ 8 (halve both numbers again); 80 ÷ 5 has the same quotient as 160 ÷ 10 (multiply both numbers by 2).

unitizing. The ability to recognize that a group of objects can be considered as a single entity. For example, 10 objects can be considered as one group of 10.

whole number. Any of the numbers 0, 1, 2, 3, 4, ...

zero property of multiplication. The property that any number multiplied by 0 is 0.
Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.
Number Sense and Numeration, Grades 4 to 6

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INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of The Ontario Curriculum, Grades 1–8: Mathematics, 2005. This guide provides teachers with practical applications of the principles and theories that are elaborated on in A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006.

The guide comprises the following volumes:
• Volume 1: The Big Ideas
• Volume 2: Addition and Subtraction
• Volume 3: Multiplication
• Volume 4: Division
• Volume 5: Fractions
• Volume 6: Decimal Numbers

The present volume – Volume 2: Addition and Subtraction – provides:
• a discussion of mathematical models and instructional strategies that support student understanding of addition and subtraction;
• sample learning activities dealing with addition and subtraction for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by “eLearning modules” that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pages 51, 68, and 80).
Relating Mathematics Topics to the Big Ideas

The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the “big ideas” of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005, p. 4)

In planning mathematics instruction, teachers generally develop learning opportunities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- operational sense
- relationships
- representation
- proportional reasoning

Each of the big ideas is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

6 Number Sense and Numeration, Grades 4 to 6 – Volume 2
The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

**Problem Solving:** Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

**Reasoning and Proving:** The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

**Reflecting:** Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

**Selecting Tools and Computational Strategies:** Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.
Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students’ own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The table on pp. 9–10 outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.
## Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intellectual</td>
<td>Generally, students in the junior grades: prefer active learning experiences that allow them to interact with their peers; are curious about the world around them; are at a concrete operational stage of development, and are often not ready to think abstractly; enjoy and understand the subtleties of humour.</td>
<td>The mathematics program should provide: learning experiences that allow students to actively explore and construct mathematical ideas; learning situations that involve the use of concrete materials; opportunities for students to see that mathematics is practical and important in their daily lives; enjoyable activities that stimulate curiosity and interest; tasks that challenge students to reason and think deeply about mathematical ideas.</td>
</tr>
<tr>
<td>Physical</td>
<td>Generally, students in the junior grades: experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); are concerned about body image; are active and energetic; display wide variations in physical development and maturity.</td>
<td>The mathematics program should provide: opportunities for physical movement and hands-on learning; a classroom that is safe and physically appealing.</td>
</tr>
<tr>
<td>Psychological</td>
<td>Generally, students in the junior grades: are less reliant on praise but still respond well to positive feedback; accept greater responsibility for their actions and work; are influenced by their peer groups.</td>
<td>The mathematics program should provide: ongoing feedback on students’ learning and progress; an environment in which students can take risks without fear of ridicule; opportunities for students to accept responsibility for their work; a classroom climate that supports diversity and encourages all members to work cooperatively.</td>
</tr>
<tr>
<td>Social</td>
<td>Generally, students in the junior grades: are less egocentric, yet require individual attention; can be volatile and changeable in regard to friendship, yet want to be part of a social group; can be talkative; are more tentative and unsure of themselves; mature socially at different rates.</td>
<td>The mathematics program should provide: opportunities to work with others in a variety of groupings (pairs, small groups, large group); opportunities to discuss mathematical ideas; clear expectations of what is acceptable social behaviour; learning activities that involve all students regardless of ability.</td>
</tr>
</tbody>
</table>

(continued)
### Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moral and ethical</td>
<td>Generally, students in the junior grades:</td>
<td>The mathematics program should provide:</td>
</tr>
<tr>
<td></td>
<td>• develop a strong sense of justice and fairness;</td>
<td>• learning experiences that provide equitable opportunities for participation by all students;</td>
</tr>
<tr>
<td></td>
<td>• experiment with challenging the norm and ask “why” questions;</td>
<td>• an environment in which all ideas are valued;</td>
</tr>
<tr>
<td></td>
<td>• begin to consider others’ points of view.</td>
<td>• opportunities for students to share their own ideas and evaluate the ideas of others.</td>
</tr>
</tbody>
</table>

(Adapted, with permission, from Making Math Happen in the Junior Grades. Elementary Teachers’ Federation of Ontario, 2004.)
Learning about addition and subtraction in the junior grades

Introduction

Instruction in the junior grades should help students to extend their understanding of addition and subtraction concepts, and allow them to develop flexible computational strategies for adding and subtracting multidigit whole numbers and decimal numbers.

Prior Learning

In the primary grades, students develop an understanding of part-whole concepts – they learn that two or more parts can be combined to create a whole (addition), and that a part can be separated from a whole (subtraction).

Young students use a variety of strategies to solve addition and subtraction problems. Initially, students use objects or their fingers to model an addition or subtraction problem and to determine the unknown amount. As students gain experience in solving addition and subtraction problems, and as they gain proficiency in counting, they make a transition from using direct modelling to using counting strategies. Counting on is one such strategy: When two sets of objects are added together, the student does not need to count all the objects in both sets, but instead begins with the number of objects in the first set and counts on from there.

"7... 8... 9... 10... 11.
There are 11 cubes altogether."
As students learn basic facts of addition and subtraction, they use this knowledge to solve problems, but sometimes they need to revert to direct modelling and counting to support their thinking. Students learn certain basic facts, such as doubles (e.g., 3 + 3 and 6 + 6), before others, and they can use such known facts to derive answers for unknown facts (e.g., 3 + 4 is related to 3 + 3; 6 + 7 is related to 6 + 6).

By the end of Grade 3, students add and subtract three-digit numbers using concrete materials and algorithms, and perform mental computations involving the addition and subtraction of two-digit numbers.

In the primary grades, students also develop an understanding of properties related to addition and subtraction:

- **Identity property:** Adding 0 to or subtracting 0 from any number does not affect the value of the number (e.g., 6 + 0 = 6; 11 – 0 = 11).
- **Commutative property:** Numbers can be added in any order, without affecting the sum (e.g., 2 + 4 = 4 + 2).
- **Associative property:** The numbers being added can be regrouped in any way without changing the sum (e.g., 7 + 6 + 4 = 6 + 4 + 7).

It is important for teachers of the junior grades to recognize the addition and subtraction concepts and skills that their students developed in the primary grades – these understandings provide a foundation for further learning in Grades 4, 5, and 6.

**KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES**

In the junior grades, instruction should focus on developing students’ understanding of meaningful computational strategies for addition and subtraction, rather than on having students memorize the steps in algorithms.

The development of computational strategies for addition and subtraction should be rooted in meaningful experiences (e.g., problem-solving contexts, investigations). Students should have opportunities to develop and apply a variety of strategies, and to consider the appropriateness of strategies in various situations.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to addition and subtraction, listed in the following table.
Learning About Addition and Subtraction in the Junior Grades

| Curriculum Expectations Related to Addition and Subtraction, Grades 4, 5, and 6 |
|---|---|---|
| By the end of Grade 4, students will: | By the end of Grade 5, students will: | By the end of Grade 6, students will: |
| **Overall Expectation** | **Overall Expectation** | **Overall Expectation** |
| • solve problems involving the addition, subtraction, multiplication, and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths, and money amounts, using a variety of strategies. | • solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies. | • solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies. |
| **Specific Expectations** | **Specific Expectations** | **Specific Expectations** |
| • add and subtract two-digit numbers, using a variety of mental strategies; | • add and subtract decimal numbers to tenths, using concrete materials and student-generated algorithms; | • add and subtract decimal numbers to thousandths, including money amounts, using concrete materials, estimation, and algorithms; |
| • solve problems involving the addition and subtraction of four-digit numbers, using student-generated algorithms and standard algorithms; | • add and subtract money amounts by making simulated purchases and providing change for amounts up to $100, using a variety of tools; | • use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution. |
| • add and subtract decimal numbers to hundredths, using concrete materials and student-generated algorithms; | • add and subtract money amounts by making simulated purchases and providing change for amounts up to $100, using a variety of tools; | • use estimation when solving problems involving the addition and subtraction of whole numbers and decimals, to help judge the reasonableness of a solution. |
| • use estimation when solving problems involving the addition, subtraction, and multiplication of whole numbers, to help judge the reasonableness of a solution. | • use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution. | |

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)

The following sections explain content knowledge related to addition and subtraction concepts in the junior grades, and provide instructional strategies that help students develop an understanding of these operations. Teachers can facilitate this understanding by helping students to:

- solve a variety of problem types;
- relate addition and subtraction;
- model addition and subtraction;
- extend knowledge of basic facts;
- develop a variety of computational strategies;
- develop estimation strategies;
- add and subtract decimal numbers.
Solving a Variety of Problem Types

Solving different types of addition and subtraction problems allows students to think about the operations in different ways. There are four main types of addition and subtraction problems: joining, separating, comparing, and part-part-whole.

A **joining** problem involves increasing an amount by adding another amount to it. The situation involves three amounts: a start amount, a change amount (the amount added), and a result amount. A joining problem occurs when one of these amounts is unknown.

Examples:
- Gavin saved $14.50 from his allowance. His grandmother gave him $6.75 for helping her with some chores. How much money does he have altogether? (Result unknown)
- There were 127 students from the primary grades in the gym for an assembly. After the students from the junior grades arrived, there were 300 students altogether. How many students from the junior grades were there? (Change unknown)
- The veterinarian told Camilla that the mass of her puppy increased by 3.5 kg in the last month. If the puppy weighs 35.6 kg now, what was its mass a month ago? (Start unknown)

A **separating** problem involves decreasing an amount by removing another amount. The situation involves three amounts: a start amount, a change amount (the amount removed), and a result amount. A separating problem occurs when one of these amounts is unknown.

Examples:
- Damian earned $21.25 from his allowance and helping his grandmother. If he spent $12.45 on comic books, how much does he have left? (Result unknown)
- There were 300 students in the gym for the assembly. Several classes went back to their classrooms, leaving 173 students in the gym. How many students returned to their classrooms? (Change unknown)
- Tika drew a line on her page. The line was longer than she needed it to be, so she erased 2.3 cm of the line. If the line she ended up with was 8.7 cm long, what was the length of the original line she drew? (Start unknown)

A **comparing** problem involves the comparison of two quantities. The situation involves a smaller amount, a larger amount, and the difference between the two amounts. A comparing problem occurs when the smaller amount, the larger amount, or the difference is unknown.

Examples:
- Antoine collected $142.15 in pledges for the read-a-thon, and Emma collected $109.56. How much more did Antoine collect in pledges? (Difference unknown)
- Boxes of Goodpick Toothpicks come in two different sizes. The smaller box contains 175 toothpicks, and the larger box contains 225 more. How many toothpicks are in the larger box? (Larger quantity unknown)
• Evan and Liddy both walk to school. Liddy walks 1.6 km farther than Evan. If Liddy’s walk to school is 3.4 km, how far is Evan’s walk? (Smaller quantity unknown)

A part-part-whole problem involves two parts that make the whole. Unlike joining and separating problems, there is no mention of adding or removing amounts in the way that a part-part-whole problem is worded. A part-part-whole problem occurs when either a part or the whole is unknown.

Examples:
• Shanlee has a collection of hockey and baseball cards. She has 376 hockey cards and 184 baseball cards. How many cards are in Shanlee’s collection? (Whole unknown)
• Erik bought 3.85 kg of fruit at the market. He bought only oranges and apples. If 1.68 kg of the fruit was oranges, what was the mass of the apples? (Part unknown)

Varying the types of problem helps students to recognize different kinds of addition and subtraction situations, and allows them to develop a variety of strategies for solving addition and subtraction problems.

Relating Addition and Subtraction
The relationship between part and whole is an important idea in addition and subtraction – any quantity can be regarded as a whole if it is composed of two or more parts. The operations of addition and subtraction involve determining either a part or the whole.

Students should have opportunities to solve problems that involve the same numbers to see the connection between addition and subtraction. Consider the following two problems.

“Julia’s class sold 168 raffle tickets in the first week and 332 the next. How many tickets did the class sell altogether?”

“Nathan’s class made it their goal to sell 500 tickets. If the students sold 332 the first week, how many will they have to sell to meet their goal?”

The second problem can be solved by subtracting 332 from 500. Students might also solve this problem using addition – they might think, “What number added to 332 will make 500?” Discussing how both addition and subtraction can be used to solve the same problem helps students to understand part-whole relationships and the connections between the operations.

It is important that students continue to develop their understanding of the relationship between addition and subtraction in the junior grades, since this relationship lays the foundation for algebraic thinking in later grades. When faced with an equation such as $x + 7 = 15$, students who interpret the problem as “What number added to 7 makes 15?” will also see that the answer can be found by subtracting 7 from 15.

Modelling Addition and Subtraction
In the primary grades, students learn to add and subtract by using a variety of concrete and pictorial models (e.g., counters, base ten materials, number lines, tallies, hundreds charts).
In the junior grades, teachers should provide learning experiences in which students continue to use models to develop understanding of mental and paper-and-pencil strategies for adding and subtracting multidigit whole numbers and decimal numbers.

In the junior grades, base ten materials and open number lines provide significant models for addition and subtraction.

**BASE TEN MATERIALS**

Base ten materials provide an effective model for addition because they allow students to recognize the importance of adding ones to ones, tens to tens, hundreds to hundreds, and so on. For example, to add 245 + 153, students combine like units (hundreds, tens, ones) separately and find that there are 3 hundreds, 9 tens, and 8 ones altogether. The sum is 398.

Students can also use base ten blocks to demonstrate the processes involved in regrouping. Students learn that having 10 or more ones requires that each group of 10 ones be grouped to form a ten (and that 10 tens be regrouped to form a hundred, and so on). After combining like base ten materials (e.g., ones with ones, tens with tens, hundreds with hundreds), students need to determine whether the quantity is 10 or greater and, if so, regroup the materials appropriately.

Concepts about regrouping are important when students use base ten materials to subtract. To solve 326 – 184, for example, students could represent 326 by using the materials like this:
To begin the subtraction, students might remove 4 ones, leaving 2 ones. Next, students might want to remove 8 tens but find that there are only 2 tens available. After exchanging 1 hundred for 10 tens (resulting in 12 tens altogether), students are able to remove 8 tens, leaving 4 tens. Finally, students remove 1 hundred, leaving 1 hundred. Students examine the remaining pieces to determine the answer: 1 hundred, 4 tens, 2 ones is 142.

Because base ten materials provide a concrete representation of regrouping, they are often used to develop an understanding of algorithms. (See Appendix 10–1 in Volume 5 of A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6 for a possible approach for developing understanding of the standard algorithm by using base ten materials.) However, teachers should be aware that some students may use base ten materials to model an operation without fully understanding the underlying concepts. By asking students to explain the processes involved in using the base ten materials, teachers can determine whether students understand concepts about place value and regrouping, or whether students are merely following procedures mechanically, without fully understanding.

OPEN NUMBER LINES

Open number lines (number lines on which only significant numbers are recorded) provide an effective model for representing addition and subtraction strategies. Showing computational steps as a series of “jumps” (drawn by arrows on the number line) allows students to visualize the number relationships and actions inherent in the strategies.

In the primary grades, students use open number lines to represent simple addition and subtraction operations. For example, students might show 36 + 35 as a series of jumps of 10’s and 1’s.

In the junior grades, open number lines continue to provide teachers and students with an effective tool for modelling various addition and subtraction strategies. For example, a student might explain a strategy for calculating 226 – 148 like this:

“I knew that I needed to find the difference between 226 and 148. So I started at 148 and added on 2 to get to 150. Next, I added on 50 to get to 200. Then I added on 26 to get to 226. I figured out the difference between 226 and 148 by adding 2 + 50 + 26.

The difference is 78.”
The teacher, wanting to highlight the student’s method, draws an open number line on the board and represents the numbers the student added on to 148 as a series of jumps.

By using a number line to illustrate the student’s thinking, the teacher gives all students in the class access to a visual representation of a particular strategy. Representing addition and subtraction strategies on a number line also helps students to develop a sense of quantity, by thinking about the relative position of numbers on a number line.

Students can also use open number lines as a tool in problem solving. For example, the teacher might have students solve the following problem.

“I am reading a very interesting novel. Last weekend, I read 198 pages. I noticed that there are 362 pages in the book. How many more pages do I have to read?”

The teacher encourages students to solve the problem in a way that makes sense to them. Some students interpret the problem as the distance between 198 and 362, and they choose to use an open number line to solve the problem. One student works with friendly numbers – making a jump of 2 to get from 198 to 200, a jump of 100 to get from 200 to 300, and a jump of 62 to get from 300 to 362. The student then adds the jumps to determine that the distance between 198 and 362 is 164.

**SELECTING APPROPRIATE MODELS**

Although base ten materials and open number lines are powerful models to help students add and subtract whole numbers and decimal numbers, it is important for teachers to recognize that these are not the only models available. At times, a simple diagram is effective in demonstrating a particular strategy. For example, to calculate 47 + 28, the following diagram shows how numbers can be decomposed into parts, then the parts added to calculate partial sums, and then the partial sums added to calculate the final sum.
Teachers need to consider which models are most effective in demonstrating particular strategies. Whenever possible, more than one model should be used so that students can observe different representations of a strategy. Teachers should also encourage students to demonstrate their strategies in ways that make sense to them. Often, students create diagrams of graphic representations that help them to clarify their own strategies and allow them to explain their methods to others.

**Extending Knowledge of Basic Facts**

In the primary grades, students develop fluency in adding and subtracting one-digit numbers, and apply this knowledge to adding and subtracting multiples of 10 (e.g., \(2 + 6 = 8\), so \(20 + 60 = 80\)).

Teachers can provide opportunities for students to explore the impact of adding and subtracting numbers that are multiples of 10, 100, and 1000 – such as 40, 200, and 5000. For example, teachers might have students explain their answers to questions such as the following:

- “What number do you get when you add 200 to 568?”
- “If you subtract 30 from 1252, how much do you have left?”
- “What number do you get when you add 3000 to 689?”
- “What is the difference between 347 and 947?”

It is important for students to develop fluency in calculating with multiples of 10, 100, and 1000 in order to develop proficiency with a variety of addition and subtraction strategies.

**Developing a Variety of Computational Strategies**

In the primary grades, students learn to add and subtract by using a variety of mental strategies and paper-and-pencil strategies. They use models, such as base ten materials, to help them understand the procedures involved in addition and subtraction algorithms.

In the junior grades, students apply their understanding of computational strategies to determine sums and differences in problems that involve multidigit whole numbers and decimal numbers. Given addition and subtraction problems, some students may tend to use a standard algorithm and carry out the procedures mechanically – without thinking about number meaning in the algorithm. As such, they have little understanding of whether the results in their computations are reasonable.

It is important that students develop a variety of strategies for adding and subtracting. If students develop skill in using only standard algorithms, they are limited to paper-and-pencil strategies that are often inappropriate in many situations (e.g., when it is more efficient to calculate numbers mentally).

Teachers can help students develop flexible computational strategies in the following ways:

- Students can be presented with a problem that involves addition or subtraction. The teacher encourages students to use a strategy that makes sense to them. In so doing, the teacher allows students to devise strategies that reflect their understanding of the problem, the
numbers contained in the problem, and the operations required to solve the problem. Student-generated strategies vary in complexity and efficiency. By discussing with the class the various strategies used to solve a problem, students can judge the effectiveness of different methods and learn to adopt these methods as their own. (The learning activities in this document provide examples of this instructional approach.)

- Teachers can help students develop skill with specific computational strategies through mini-lessons (Fosnot & Dolk, 2001a). With this approach, students are asked to solve a sequence of related computations – also called a “string” – which allows students to understand how a particular strategy works. (In this volume, see Appendix 2–1: Developing Computational Strategies Through Mini-Lessons for more information on mini-lessons with math strings.)

The effectiveness of these instructional methods depends on students making sense of the numbers and working with them in flexible ways (e.g., by decomposing numbers into parts that are easier to calculate). Learning about various strategies is enhanced when students have opportunities to visualize how the strategies work. By representing various methods visually (e.g., drawing an open number line that illustrates a strategy), teachers can help students understand the processes used to add and subtract numbers in flexible ways.

**ADDITION STRATEGIES**

This section explains a variety of addition strategies. Although the examples provided often involve two- or three-digit whole numbers, it is important that the number size in problems aligns with the grade-level curriculum expectations and is appropriate for the students’ ability level.

The examples also include visual representations (e.g., diagrams, number lines) of the strategies. Teachers can use similar representations to model strategies for students.

It is difficult to categorize the following strategies as either mental or paper-and-pencil. Often, a strategy involves both doing mental calculations and recording numbers on paper. Some strategies may, over time, develop into strictly mental processes. However, it is usually necessary – and helpful – for students to jot down numbers as they work through a new strategy.

**Splitting strategy:** Adding with base ten materials helps students to understand that ones are added to ones, tens to tens, hundreds to hundreds, and so on. This understanding can be applied when using a splitting strategy, in which numbers are decomposed according to place value and then each place-value part is added separately. Finally, the partial sums are added to calculate the total sum.
The splitting strategy is often used as a mental addition strategy. For example, to add 25 + 37 mentally, students might use strategies such as the following:

- add the tens first (20 + 30 = 50), then add the ones (5 + 7 = 12), and then add the partial sums (50 + 12 = 62); or
- add the ones first (5 + 7 = 12), then add the tens (20 + 30 = 50), and then add the partial sums (12 + 50 = 62).

The splitting strategy is less effective for adding whole numbers with four or more digits (and with decimal numbers to hundredths and thousandths), because adding all the partial sums takes time, and students can get frustrated with the amount of adding required.

**Adding-on strategy:** With this strategy, one addend is kept intact, while the other addend is decomposed into friendlier numbers (often according to place value – into ones, tens, hundreds, and so on). The parts of the second addend are added onto the first addend. For example, to add 36 + 47, students might:

- add the first addend to the tens of the second addend (36 + 40 = 76), and then add on the ones of the second addend (76 + 7 = 83);
- add the first addend to the ones of the second addend (36 + 7 = 43), and then add on the tens of the second addend (43 + 40 = 83).

The adding-on strategy can be modelled using an open number line. The following example shows 346 + 125. Here, 125 is decomposed into 100, 20, and 5.

The adding-on strategy can also be applied to adding decimal numbers. To add 8.6 + 5.4, for example, students might add 8.6 + 5 first, and then add 13.6 + 0.4. The following number line illustrates the strategy.

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The learning about addition and subtraction in the junior grades.
Moving strategy: A moving strategy involves “moving” quantities from one addend to the other to create numbers that are easier to work with. This strategy is particularly effective when one addend is close to a friendly number (e.g., a multiple of 10). In the following example, 296 is close to 300. By “moving” 4 from 568 to 296, the addition question can be changed to 300 + 564.

The preceding example highlights the importance of examining the numbers in a problem in order to select an appropriate strategy. A splitting strategy or an adding-on strategy could have been used to calculate 296 + 568; however, in this case, these strategies would be cumbersome and less efficient than a moving strategy.

Compensation strategy: A compensation strategy involves adding more than is needed, and then taking away the extra at the end. This strategy is particularly effective when one addend is close to a friendly number (e.g., a multiple of 10). In the following example, 268 + 390 is solved by adding 268 + 400, and then subtracting the extra 10 (the difference between 390 and 400).

268 + 390
268 + 400 = 668
668 − 10 = 658

A number line can be used to model this strategy.

SUBTRACTION STRATEGIES

The development of subtraction strategies is based on two interpretations of subtraction:

• Subtraction can be thought of as the distance or difference between two given numbers. On the following number line showing 256 − 119, the difference (137) is the space between 119 and 256. Thinking about subtraction as the distance between two numbers is evident in the adding-on strategy described below.

256 − 119
119 + 137
137 is added to 119 to get to 256.

Number Sense and Numeration, Grades 4 to 6 – Volume 2
Subtraction can be thought of as the removal of a quantity from another quantity. On the following number line, the difference is found by removing (taking away) 119 from 256. Thinking about subtraction as taking away between two numbers is evident in the partial-subtraction strategy and the compensation strategy described below.

Adding-on strategy: This strategy involves starting with the smaller quantity and adding on numbers until the larger quantity is reached. The sum of the numbers that are added on represent the difference between the larger and the smaller quantities. The following example illustrates how an adding-on strategy might be used to calculate 634 – 318:

Another version of the adding-on strategy involves adding on to get to a friendly number first, and then adding hundreds, tens, and ones. For example, students might calculate 556 – 189 by:
- adding 11 to 189 to get to 200; then
- adding 300 to 200 to get to 500; then
- adding 56 to 500 to get to 556; then
- adding the subtotals, 11 + 300 + 56 = 367. The difference between 556 and 189 is 367.

A number line can be used to model the thinking behind this strategy.
An adding-on strategy can also be used to solve subtraction problems involving decimal numbers. For example, the following number line shows 5.32 – 2.94.

In this example, 0.06, 2.0, and 0.32 are added together to calculate the difference between 5.32 and 2.94. (0.06 + 2.0 + 0.32 = 2.38)

With an adding-on strategy, students need to keep track of the quantities that are added on. Students might use pencil and paper to record the numbers that are added on, or they might keep track of the numbers mentally.

Partial-subtraction strategy: With a partial-subtraction strategy, the number being subtracted is decomposed into parts, and each part is subtracted separately. In the following example, 325 is decomposed according to place value (into hundreds, tens, and ones).

The number being subtracted can also be decomposed into parts that result in a friendly number, as shown below.

Compensation strategy: A compensation strategy for subtraction involves subtracting more than is required, and then adding back the extra amount. This strategy is particularly effective when the number being subtracted is close to a friendly number (e.g., a multiple of 10). In the following example, 565 – 285 is calculated by subtracting 300 from 565, and then adding back 15 (the difference between 285 and 300).

Modelled on the number line, compensation strategies look like big jumps backwards, and then small jumps forward:
Constant-difference strategy: An effective strategy for solving subtraction problems mentally is based on the idea of a constant difference. Constant difference refers to the idea that the difference between two numbers does not change after adding or subtracting the same quantity to both numbers. In the following example, the difference between 290 and 190 is 100. Adding 10 to both numbers does not change the difference -- the difference between 300 and 200 is still 100.

This strategy can be applied to subtraction problems. For example, a student might solve a problem involving 645 – 185 in the following way:

"If I add 15 to 185, it becomes 200, which is an easy number to subtract. But I have to add 15 to both numbers, so the question becomes 660 – 200, which is 460."

A constant-difference strategy usually involves changing the number being subtracted into a friendlier number. As such, the strategy is useful in subtraction with decimal numbers, especially in problems involving tenths. To solve 15.1 – 3.2, for example, 0.2 could be subtracted from both 15.1 and 3.2 to change the problem to 14.9 – 3.0. The subtraction of a whole-number value (3.0), rather than the decimal number in the original problem, simplifies the calculation. The example is illustrated on the following number line.

SELECTING AN APPROPRIATE STRATEGY

As with all computational strategies, students should first examine the numbers in the problem before choosing a strategy. Removing hundreds, tens, and ones does not always work neatly with regrouping. For example, to calculate 731 – 465, a partial-subtraction strategy of subtracting 400, 60, and 5 is not necessarily an efficient strategy because of the regrouping required to subtract 6 tens from 3 tens. However, an adding-on strategy might be used: Add 35 to 465 to get to 500, add 200 to get to 700, add 31 to get to 731, and add 35 + 200 + 31 to calculate a total difference of 266. A constant-difference strategy could also be applied: Add 35 to both numbers to change the subtraction to 766 – 500.

Developing Estimation Strategies

It is important for students to develop skill in estimating sums and differences. Estimation is a practical skill in many real-life situations. It also provides a way for students to judge the reasonableness of a calculation performed with a calculator or on paper.
Selecting an appropriate strategy depends on the context of a given problem and on the numbers involved in the problem. Consider the following situation.

“Aaron needs to buy movie tickets for $8.25, popcorn for $3.50, and a drink for $1.75. About how much money should Aaron bring to the movies?”

In this situation, students should recognize that an appropriate estimation strategy would involve rounding up each money amount to the closest whole-number value, so that Aaron has enough money.

The table below lists several estimation strategies for addition and subtraction. It is important to note that the word “rounding” is used loosely—it does not refer to any set of rules or procedures for rounding numbers (e.g., look to the number on the right, if it is greater than 5 then round up…).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Example</th>
</tr>
</thead>
</table>
| Rounding each number to the nearest multiple of 10, 100, 1000, and so on | 891 + 667 is about 900 + 700 = 1600  
891 – 667 is about 890 – 670 = 220 |
| Rounding numbers to friendly numbers                        | 891 + 667 is about 900 + 650 = 1550  
891 – 667 is about 900 – 650 = 250 |
| Rounding one number but not the other                        | 891 – 667 is about 900 + 667 = 1567 |
| Rounding one number up and the other down                   | 891 + 667 is about 900 + 660 = 1560 |
| Rounding both numbers up or both numbers down               | 891 – 667 is about 900 – 700 = 200  
891 – 667 is about 800 – 600 = 200 |
| Finding a range                                             | 538 + 294 is between 700 (500 + 200) and  
900 (600 + 300)  
418 – 126 is between 200 (400 – 200) and  
400 (500 – 100) |
| Using compatible numbers                                    | 626 + 328 is about 626 + 324 = 950  
747 – 339 is about 747 – 347 = 400 |

**Adding and Subtracting Decimal Numbers**

Many of the addition and subtraction strategies described above also apply to computations with decimal numbers. (See the preceding examples involving decimal numbers under “Splitting strategy”, “Adding-on strategy” for addition, “Adding-on strategy” for subtraction, and “Constant-difference strategy”.)
Using the standard algorithm is a practical strategy for adding and subtracting decimal numbers in many situations. The standard algorithm is impractical if the calculations can easily be performed mentally, or if the problem involves numbers that are best calculated using a calculator. When teaching addition and subtraction with decimal numbers, teachers should develop strategies through problem-solving situations and strive to create meaningful contexts for the operations. For example, problems involving money expressed as decimal numbers provide contexts that can be relevant to students. As well, measurement problems (e.g., involving length or mass) often involve working with decimal numbers.

Perhaps the most difficult challenge students face with decimal-number computation is adding or subtracting numbers that do not share a common “end point” (e.g., adding tenths to thousandths, subtracting hundredths from tenths). Part of the difficulty arises from the lack of contextual referents – rarely are people called on in real-life situations to add or subtract numbers like 18.6, 125.654, and 55.26 in the same situational context.

It is more important that teachers emphasize place-value concepts when they help their students understand decimal-number computations by using algorithms. Rather than simply following the rule of “lining up the decimals” in an algorithm, students should recognize that like-units need to be added or subtracted – ones added to or subtracted from ones, tenths to and from tenths, hundredths to and from hundredths, and so on. With an understanding of place value in an algorithm, students recognize that annexing zeros to the decimal part of a number does not change the value of the number. The following example shows how an addition expression can be rewritten by including zeros in the hundredths and thousandths places in one of the addends.

\[
\begin{align*}
18.6 & \quad 18.600 \\
+125.654 & \quad +125.654 \\
\end{align*}
\]

Estimation plays an important role when adding and subtracting decimal numbers using algorithms. For example, students can recognize that 34.96 – 29.04 is close to 35 – 30, and estimate that the difference will be about 5. After completing the algorithm, students can refer back to their estimate to determine whether the result of their calculation is reasonable.

A Summary of General Instructional Strategies

Students in the junior grades benefit from the following instructional strategies:

- solving a variety of addition and subtraction problems, including joining, separating, comparing, and part-part-whole problems;
- using concrete and pictorial models, such as base ten materials and open number lines, to develop an understanding of addition and subtraction concepts and strategies;
- providing opportunities to connect subtraction to addition through problem solving;

Learning About Addition and Subtraction in the Junior Grades
solving addition and subtraction problems that serve different instructional purposes (e.g., to introduce new concepts, to learn a particular strategy, to consolidate ideas);

• providing opportunities to develop and practise mental computation and estimation strategies.

The Grades 4–6 Addition and Subtraction module at www.eworkshop.on.ca provides additional information on developing addition and subtraction concepts with students. The module also contains a variety of learning activities and teaching resources.
APPENDIX 2-1: DEVELOPING COMPUTATIONAL STRATEGIES THROUGH MINI-LESSONS

Introduction

“A number of researchers have argued that mental arithmetic . . . can lead to deeper insights into the number system”

(Kilpatrick, Swafford, & Findell, 2001, p. 214).

Developing efficient mental computational strategies is an important part of mathematics in the junior grades. Students who learn to perform mental computations develop confidence in working with numbers and are able to explore more complex mathematical concepts without being hindered by computations.

Mini-Lessons With Mental Math Strings

One method for developing mental computational skills is through the use of mini-lessons – short, 10- to 15-minute lessons that focus on specific computational strategies (Fosnot & Dolk, 2001a). Unlike student-centred investigations, mini-lessons are more teacher-guided and explicit. Each mini-lesson is designed to develop or “routinize” a particular mental math strategy.

A computational mini-lesson often involves a “string” – a structured sequence of four to seven related computations that are designed to elicit a particular mental computational strategy. The following is an example of a string that focuses on a compensation strategy for addition. This strategy involves adding more than is needed (often a multiple of 10) and then subtracting the extra amount.

\[
\begin{align*}
46 + 10 & \\
46 + 9 & \\
64 + 20 & \\
64 + 19 & \\
36 + 19 & 
\end{align*}
\]
The computations in this string are related to one another. Students know the answer to 46 + 10, and they also know that 46 + 9 is one less than 46 + 10. The third computation, 64 + 20, is like the first, only this time students are adding 20 instead of 10. They can calculate 64 + 19 by knowing that the answer is one less than 64 + 20. The last computation, 36 + 19, has no “helper” (e.g., 46 + 10 and 64 + 20, shown in bold type, are “helper” computations for 46 + 9 and 64 + 19). However, the previous four computations follow a pattern that helps students to apply a compensation strategy. Students might consider 36 + 19 and think: “36 + 20 = 56. But 36 + 19 is one less, so 36 + 19 = 55.”

A mini-lesson usually proceeds in the following way:

- The teacher writes the first computation horizontally on the board and asks students to calculate the answer.
- Students are given time to calculate mentally. Students may jot down numbers on paper to help them keep track of figures, but they should not perform paper-and-pencil calculations that can be done mentally.
- The teacher asks a few students to explain how they determined the answer.
- The teacher models students’ thinking on the board by using diagrams, such as open number lines, to illustrate various strategies.
- The teacher presents the remaining computations, one at a time. Strategies for each computation are discussed and modelled.
- After all computations have been solved, the focus strategy is identified and discussed.

Following the mini-lesson, the teacher should reflect on the effectiveness of the string in helping students to develop an understanding of the focus strategy. Reflecting on the mini-lesson will help to provide direction for future lessons. For example, the teacher may realize that students are not ready for a particular strategy and that they need more experience with a related concept first. Or the teacher might determine that students use a strategy effectively and are ready to learn a new one.

Mini-lessons can be used throughout the year, even when the main mathematics lesson deals with concepts from other strands of mathematics. Mini-lessons can take place before the regular math lesson or at any other time during the day.

In a mini-lesson, teachers might also pose an individual computation instead of strings. This approach encourages students to examine the numbers in the expression in order to determine an appropriate strategy (rather than looking at the helper computations to determine a strategy). The various strategies used by students are discussed and modelled.

Note: To develop confidence in teaching computational strategies with strings, teachers might work with a small group of students before they use mini-lessons with the whole class.
Modelling Student Thinking and Strategies

It is important for teachers to encourage students to communicate their thinking when they perform computations during mini-lessons. When students explain their thinking, they clarify their strategies for themselves and their classmates, and they make connections between different strategies. During mini-lessons, the teacher records student thinking on the board in order to demonstrate various strategies for the class.

The open number line provides an effective model for representing students’ thinking and strategies. For example, a student might explain how he determined the answer to 64 + 20 in the string given above like this:

“Well, I started at 64, and then I added on 10's . . . 64 and 10 is 74, and 74 plus 10 is 84. So 64 + 20 is 84.”

The teacher could illustrate the student’s strategy by drawing a number line on the board.

Later, in the mini-lesson, another student might explain how she solved 36 + 19:

“I added 36 + 20 and got 56, but I knew that was too much because I was adding 19 and not 20, so I had to go back 1 to 55.”

The teacher’s drawing of a number line helps the class understand the student’s thinking.

The modelling of students’ thinking helps the class to visualize strategies that might not be clearly understood if only oral explanations of those strategies are given. Recorded models also allow students to develop a mental image of different strategies. These images can help students to reason towards a solution when presented with other computations.

A Mini-Lesson in Action

The following scenario provides a description of a mini-lesson with a Grade 4 class. The teacher wants to highlight a compensation strategy for subtraction. This strategy involves subtracting more than is needed (often a multiple of 10), and then adding back the extra amount. In this lesson, the teacher uses the string shown at right. She developed the string prior to the lesson, putting considerable thought into developing a sequence of questions that highlight the focus strategy.
The teacher begins the mini-lesson by writing the first computation, 50 – 10, on the board. She asks, “Who knows the answer to this question? Show me a thumbs-up when you know it.” Most students know the answer right away. Devon responds: “40. I subtracted 1 from the 5 to get 4, so the answer is 40.” The teacher asks, “Really? When I subtract 1 from 5, I get 4—not 40. How did you get 40?” Devon clarifies that the 1 he subtracted was actually a 10, since it was in the tens column. The teacher draws an open number line to show the jump backwards from 50 to 40.

Next, the teacher writes 50 – 20 on the board and again most students show their thumbs quickly. Keri offers her solution: “30. I just jumped backwards another 10.” The teacher models Keri’s thinking on the board by using an open number line.

When the teacher writes 50 – 19 on the board, the students are pensive, and only a few quickly offer a thumbs-up. She gives the class time to think about the question. “Who knows this one?” Laura answers “31”, and the teacher asks her to explain how she figured out the answer. “Well, on the second question we started at 50 and jumped back 20. That got us to 30. But for this one, I didn’t have to jump back 20, I only needed to jump back 19, so I added 1 when I was done.” “When you were done?” asks the teacher. “Yeah, when I was done jumping 20.” The teacher gives the class some time to think about this, and then asks if anyone can explain Laura’s strategy. Moira says, “I know what Laura was trying to do, but I don’t get it. I know the answer is 31. I made 19 into 20, and then took 20 away from 50 to get 30. Then I added one more to get 31, but I don’t get it.” The teacher asks, “What don’t you get?” “Well, if I added 1 to 19 to make 20, shouldn’t I take it away at the end? That would give me 29, not 31.”
The teacher models Moira’s idea.

Moira is making a connection to a compensation strategy for addition that she is comfortable using. (To add 56 + 29, 1 is added to 29 to make a friendly number of 30. At the end, she must compensate – she needs to take away the extra amount that was added to make the friendly number.)

Moira wonders aloud why her strategy would give the wrong answer. The teacher asks the class to consider Moira’s question. Very few hands go up, and she wonders whether most students follow the discussion. After a while, Dennis thinks he has the answer to Moira’s question.

“It’s like this, Moira. You took 1 from somewhere to make 19 into 20. Now you have to put it back. If you take it away at the end, you’re taking away 21, not 19. You’re only supposed to take away 19, but 20 is easier, so you borrowed 1 from somewhere to take away 20. At the end you put it back.”

The teacher draws another open number line.

Dennis’s explanation makes sense to many of the students. Moira sums up his explanation nicely. “It’s like Dennis said – if I take away 20, I’m taking too much, so at the end I have to put some back.”

The teacher continues with the string, and the students calculate 75 – 20, 75 – 19, and 87 – 18. The class discusses strategies, and the teacher models the ideas on the board.

When she writes the final computation, 145 – 28, the extra digit intimidates some students. “Whoa, now hundreds? We can’t do this mentally.”

Izzy determines the answer by decomposing 28 into 20 + 3 + 5: “145 minus 5 is 140, take away 3 is 137. Then I took 20 away to get 117.”

Izzy’s strategy is effective and efficient, although it is not as efficient as a compensation strategy that involves subtracting 30 and adding back 2.

Appendix 2–1: Developing Computational Strategies Through Mini-Lessons

(continued)
Dennis refers back to his strategy. “Well, you could use ‘put-it-back’ too. 145 minus 30 is 115. Then because I only needed to take away 28, I add the 2 back at the end, so the answer is 117.”

The teacher asks, “Did you jump 30 all at once, or make jumps of 10?”

“Jumps of 10,” answers Dennis.

At the conclusion of the mini-lesson, the teacher recognizes that only a few students can confidently use the compensation strategy for subtraction. She observed that many students “jump back” by 10’s, as did Dennis, rather than subtract a multiple of 10 (e.g., students think “145 – 10 – 10 – 10”, rather than “145 – 30”). She decides to focus on a strategy that involves subtracting multiples of 10 in the next mini-lesson.

This mini-lesson provided the teacher with valuable feedback and direction for strategies to pursue in the future. She plans to revisit this strategy when students are more confidently able to subtract multiples of 10.

The effectiveness of the mini-lesson depends on the teacher’s efforts to engage students in the activity. Specifically, the teacher:

• expects all students to try the computations in the strings;
• has students use a thumbs-up signal to show when they have completed the computation – this technique encourages all students to determine an answer;
• encourages students to explain their strategies;
• asks students to respond to others’ strategies;
• asks students to clarify their explanations for others;
• accepts and respects students’ thinking, even though their strategies may reflect misconceptions;
• poses questions that help students clarify their thinking;
• models strategies on the board so that students can “see” one another’s thinking.

Developing Strings for Addition and Subtraction

In order to design strings and plan mini-lessons effectively, teachers must have an understanding of various mental computational strategies. The following are some different strategies for mental addition and subtraction.
Strings are usually made up of pairs or groups of computations that are related. “Helper” computations (questions that students are able to answer easily) are followed by a computation that can be solved by applying the focus strategy. The following example shows the structure of a string that focuses on the adding-on strategy. In this case, the strategy involves adding the tens from the second addend first, and then adding on the ones.

### Addition Strategies

**Adding On**

With this strategy, the number being added is decomposed into parts, and each part is added separately.

- **Example**: 136 + 143
  1. 136 + 100 = 236.
  2. 236 + 40 = 276.
  3. 276 + 3 = 279.

**Compensation**

This strategy involves adding more than is required, and then subtracting the extra amount.

- **Example**: 236 + 297
  1. 236 + 300 = 536.
  2. Subtract 3 (the difference between 297 and 300):
     - 536 – 3 = 533.
     - So, 236 + 297 = 533.

**Moving**

This strategy involves “moving” a quantity from one addend to another to create an expression with friendly numbers.

- **Example**: 153 + 598
  1. Move 2 from 153 to 598.
     - 151 + 600 = 751.
     - So, 153 + 598 = 751.

### Subtraction Strategies

**Partial Subtraction**

With this strategy, the number being subtracted is decomposed into parts, and each part is subtracted separately.

- **Example**: 387 – 146
  1. 387 – 100 = 287.
  2. 287 – 40 = 247.

**Compensation**

This strategy involves subtracting more than is required, and then adding back the extra amount.

- **Example**: 547 – 296
  1. 547 – 300 = 247.
  2. Add back 4 (the difference between 296 and 300):
     - 247 + 4 = 251.
     - So, 547 – 296 = 251.

**Constant Difference**

The difference between two numbers does not change after adding or subtracting the same quantity to both numbers.

- **Example**: 146 – 38
  1. Add 2 to both numbers to create an expression with friendly numbers:
     - So, 146 – 38 = 108.

---

**Appendix 2–1: Developing Computational Strategies Through Mini-Lessons**

35
The following are examples of strings that are based on the computation strategies explained above.

<table>
<thead>
<tr>
<th>Examples of Addition Strings</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adding On</strong></td>
</tr>
<tr>
<td>47 + 20</td>
</tr>
<tr>
<td>47 + 3</td>
</tr>
<tr>
<td>47 + 23</td>
</tr>
<tr>
<td>147 + 20</td>
</tr>
<tr>
<td>147 + 25</td>
</tr>
<tr>
<td>147 + 35</td>
</tr>
<tr>
<td>341 + 36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples of Subtraction Strings</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Using Partial Subtraction</strong></td>
</tr>
<tr>
<td>85 – 20</td>
</tr>
<tr>
<td>85 – 3</td>
</tr>
<tr>
<td>85 – 23</td>
</tr>
<tr>
<td>275 – 100</td>
</tr>
<tr>
<td>275 – 40</td>
</tr>
<tr>
<td>275 – 3</td>
</tr>
<tr>
<td>275 – 143</td>
</tr>
</tbody>
</table>
Mini-lessons with strings are not intended to be a means of teaching a prescribed list of computational procedures. Rather than simply following a series of computations in a resource book, teachers should develop their own strings based on the needs of their students. In developing strings, teachers need to focus on particular computational strategies that will extend students’ skill in mental computation. Whenever possible, a string should relate to, or be an extension of, a mental strategy that students have already practised.

Careful thought should go into the development of a string. Thinking about the computations presented in a string, as well as possible student responses, allows teachers to anticipate how the mini-lesson might unfold. Teachers should consider alternative strategies students might use (strategies that are different from the intended focus strategy). Teachers need to ask themselves: Why might students come up with alternative strategies? How are these alternative strategies related to the focus strategy? How can models, such as open number lines, help students to see the relationship between different strategies?

Often, student responses determine the direction teachers should take in developing subsequent strings. If students experience difficulties in using a focus strategy, teachers should consider whether students need practise with a related, more fundamental, strategy first. As well, teachers need to consider whether the string used in the mini-lesson was well crafted and constructed, or whether other computations would have been more effective in developing the strategy.

**Strings for Multiplication and Division**

Strings used for multiplication and division are similar to those used for addition and subtraction:

- Each string focuses on a particular strategy.
- A string comprises helper computations as well as computations that can be solved by applying the focus strategy.
- Teachers should model students’ strategies to illustrate students’ thinking.

The following is an example of a multiplication string that focuses on the use of the distributive property in mental computation.

\[
\begin{align*}
8 \times 5 \\
8 \times 40 \\
8 \times 45 \\
6 \times 4 \\
6 \times 30 \\
6 \times 34 \\
5 \times 63
\end{align*}
\]
This string helps students to understand that a multiplication expression, such as $63 \times 5$, can be calculated by multiplying ones by tens ($5 \times 60 = 300$), then multiplying ones by ones ($5 \times 3 = 15$), and then adding the partial products ($300 + 15 = 315$).

The open array provides a model for demonstrating this strategy.

The open array helps students to visualize how 63 can be decomposed into 60 and 3, then each part can be multiplied by 5, and then the partial products can be added to determine the total product.

**Conclusion**

Learning mathematics is effective when it is done collaboratively among students. The same can be said for teachers as they begin to develop strings and develop computational strategies using mini-lessons. Working with other teachers allows for professional dialogue about strategies and student thinking.

Teachers can find more information on developing mini-lessons with math strings in several of the resources listed on the following page; specifically, the three *Young Mathematicians at Work* volumes by Fosnot and Dolk (2001a, 2001b, 2001c), and the books on mini-lessons by Fosnot, Dolk, Cameron, and Hersch (2004) and Fosnot, Dolk, Cameron, Hersch, and Teig (2005).
REFERENCES


Learning Activities for Addition and Subtraction

Introduction

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to addition and subtraction. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or task.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students’ understanding of mathematical concepts.
HOME CONNECTION: This section is addressed to parents or guardians, and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.
Grade 4 Learning Activity
Rising Waters

OVERVIEW
In this learning activity, students find different ways to fill a Kindergarten water table basin with exactly 25 L of water using different-sized containers. The activity provides an opportunity for students to explore different strategies for adding and subtracting decimal numbers to tenths.

BIG IDEAS
This learning activity focuses on the following big ideas:

Quantity: Students explore the “howmuchness” of decimal numbers to tenths by comparing and ordering the capacity of different containers.

Operational sense: Students apply their understanding of addition and subtraction to perform calculations with decimal numbers.

Relationships: This learning activity allows students to see the relationship between tenths and wholes (e.g., that 10 tenths makes 1 whole).

Representation: Students represent decimal amounts visually by using concrete materials, and symbolically by using decimal notation.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations. Students will:

• represent, compare, and order decimal numbers to tenths, using a variety of tools (e.g., concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings) and using standard decimal notation;

• add and subtract decimal numbers to tenths, using concrete materials (e.g., paper strips divided into tenths, base ten materials) and student-generated algorithms (e.g., “When I added 6.5 and 5.6, I took five tenths in fraction circles and added six tenths in fraction circles to give me one whole and one tenth. Then I added 6 + 5 + 1.1, which equals 12.1.”).

These specific expectations contribute to the development of the following overall expectation. Students will:

• solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.
ABOUT THE LEARNING ACTIVITY

MATERIALS
- overhead transparency of AddSub4.BLM1: What’s The Capacity?
- overhead projector
- overhead marker
- manipulatives for modelling decimal numbers to tenths (e.g., fraction circles, paper strips cut from AddSub4.BLM2: Paper Strips Divided Into Tenths, base ten blocks)
- sheets of paper (1 per pair of students)
- half sheet of chart paper or a large sheet of newsprint (1 per pair of students)
- markers (a few per pair of students)
- AddSub4.BLM3a-c: Addition and Subtraction Puzzles (1 per student)

MATH LANGUAGE
- decimal number
- tenths
- addition
- sum
- subtraction
- capacity
- litres
- friendly number
- difference

Note: To connect decimal numbers to their meaning, it is helpful to read 2.6 as “two and six tenths”, rather than as “two point six” or “two decimal six”.

INSTRUCTIONAL SEQUENCING
Before starting this learning activity, students should have had opportunities to represent, compare, and order decimal numbers to tenths using a variety of tools (e.g., fraction circles, paper strips divided into tenths, base ten blocks, number lines, drawings). This learning activity allows students to solve a problem by applying their understanding of decimal numbers and of addition and subtraction.

ABOUT THE MATH
In Grade 4 students learn to add and subtract decimal number to tenths. In this learning activity, students have an opportunity to add decimal numbers using strategies that make sense to them (e.g., using manipulatives, using drawings, using number lines, using reasoning), without having to follow the steps in an algorithm. The learning activity also provides experience in adding decimal numbers to determine friendly whole-number values (e.g., 1.1 + 2.9 = 4, 1.4 + 3.6 = 5).

GETTING STARTED
Display an overhead transparency of AddSub4.BLM1: What’s the Capacity? Draw students’ attention to the numbers on the bottom of the overhead transparency, and explain that these numbers represent the capacities of the different containers. Review the meaning of “capacity” (the greatest amount or volume that a container can hold).
Ask:
• “Which container do you think has the greatest capacity?”
• “Which capacity listed at the bottom of the transparency corresponds to this container?”
• “How do you know that 3.6 L is the greatest capacity?”

Have students work with a partner to match the other four capacities with containers. Have students explain their rationales for matching each capacity with a container. Use an overhead marker to record the capacity below each container (dish soap, 1.1 L; laundry soap, 2.9 L; water, 0.5 L; juice, 1.4 L). Explain that containers are sometimes reused to carry water to pour into other containers. Ask: “How much water would fill both the dish soap container and the juice container?”

Provide access to manipulatives (e.g., fraction circles, paper strips cut from AddSub4.BLM2: Paper Strips Divided Into Tenths, base ten blocks), and invite students to use the manipulatives to help them determine a solution. Allow approximately one minute for students to work individually, and then have them explain their strategies to a partner.

Invite a few students to share their strategies with the class. Encourage them to demonstrate their thinking using manipulatives or diagrams. For example, students might use base ten blocks or fraction strips to represent the capacity of each container (1.1 L and 1.4 L), and then combine the materials to determine the total capacity. Other students might add the two capacities mentally. A possible mental computation might involve adding 1.4 L and 1 L to get 2.4 L, and then adding 0.1 to get 2.5 L. Model addition strategies by drawing a number line on the board to help students visualize addition processes.

WORKING ON IT

Explain the problem situation.

“The Kindergarten teacher often needs to fill the water table basin with water. The basin is too heavy to carry when it is filled with water, so the teacher wants to use empty plastic containers to fill the basin. He doesn’t want to fill the basin to the top, because the water will spill over the sides when the children are playing in it. He has discovered that 25 L of water is the ideal capacity.”

Refer to the overhead transparency of AddSub4.BLM1: What’s the Capacity? and explain the problem:

“The Kindergarten teacher found 5 empty containers that he can use to fill the water basin. The containers have capacities of 0.5 L, 1.1 L, 1.4 L, 2.9 L, and 3.6 L. He can use 1, 2, 3, 4, or all 5 containers. How can he use these containers to fill the water basin with 25 L of water? Is there more than one way to fill the water basin?”
Organize students into pairs. Explain that students will work with their partner to solve the problem. Encourage them to use manipulatives (e.g., fraction circles, fraction strips, base ten blocks) or diagrams (e.g., open number lines) to help them determine a solution. Provide each pair of students with a sheet of paper on which they can record their work.

**STRATEGIES STUDENTS MIGHT USE**

**USING TRIAL AND ERROR**
Students might try different combinations of containers to reach a total of 25 L.

**USING FRIENDLY NUMBERS**
Students might find that certain combinations of containers provide whole-number capacities that are easy to work with. For example, the 3.6 L and 1.4 L containers can be added to get 5 L. Students would reason that they would need to combine five 3.6 L and five 1.4 L containers to get 25 L.

**USING REPEATED ADDITION**
Students might try to maximize the use of the largest container and repeatedly add 3.6 L six times until they get to 21.6 L. (Adding 3.6 seven times results in 25.2, a value that is greater than 25.) To get from 21.6 L to 25 L, students would add on containers that provide a capacity of 3.4 L (e.g., 2.9 L + 0.5 L).

Observe students as they work. Ask them questions about their strategies and solutions:
- “What strategy are you using to solve the problem?”
- “What is working well with your strategy? What is not working well?”
- “How can you prove that your solution is correct?”
- “Can you find another solution?”

After pairs of students have found one or more solutions, provide them with markers and a half sheet of chart paper or a large sheet of newsprint. Ask students to show how they solved the problem in a way that can be clearly understood by the Kindergarten teacher. Encourage them to use diagrams (e.g., open number lines) to show their thinking.

**REFLECTING AND CONNECTING**
Have pairs of students present their solutions to the class. Try to include a variety of strategies, solutions, and formats. Have each pair justify their solution(s) by asking them to prove that each combination of containers would provide 25 L of water. Encourage students to clarify their understanding of presented solutions and strategies by asking questions such as:
- “What strategy did the presenters use to determine the total number of litres?”
- “Why do you think the presenters used that strategy?”
- “What questions about the strategy or solution do you have for the presenters?”
Post students’ work in the classroom following each presentation. After several pairs have explained their strategies, ask:

• “Which solutions would you recommend to the Kindergarten teacher? Why?”
• “Which solutions would you not recommend? Why?”

(Students might assess the suitability of different solutions by considering the number of times containers need to be filled.)

Draw students’ attention to the different formats used to record solutions. Ask questions such as:

• “In what different ways did pairs record their strategies and solutions?”
• “Which forms are easy to understand? Why is the work clear and easy to understand?”

ADAPTATIONS/EXTENSIONS

Simplify the problem for students who experience difficulties by reducing the capacity of the water table basin to 10 L and/or reducing the number of containers. Prompt students to search for friendly numbers by asking:

• “What combination of containers would create a friendly number?”
• “How can you use this friendly number to get close to the capacity of the basin?”

Extend the activity for students requiring a greater challenge by posing the following problems:

• “What is the fewest number of containers that you could use to fill the basin? How many times would you need to fill each container?”
• “What is the greatest number of containers that you could use? How many times would you need to fill each container?”

ASSESSMENT

Have students, individually, solve the following problem. Ask students to record their solutions, reminding them to show their ideas in a way that can be clearly understood by others.

“How can you fill a bucket with 20 L of water using any of these five containers with the following capacities: 0.8 L, 1.3 L, 1.5 L, 2.2 L, 2.7 L? You may use one or more of the containers in your solution.”

Encourage students to use manipulatives (e.g., fraction circles, fraction strips, base ten blocks) or diagrams (e.g., open number lines) to help them determine a solution. Observe students’ work to assess how well they:

• add the capacities of containers using manipulatives and/or diagrams;
• solve the problem by finding a combination of containers with a capacity of 20 L;
• demonstrate their thinking using materials, drawings, and/or written explanations;
• explain their strategy and solution.
HOME CONNECTION

Send home *AddSub4.BLM3a–c: Addition and Subtraction Puzzles*. In this Home Connection activity, students and parents solve puzzles involving the addition and subtraction of decimal numbers. Include a copy of *AddSub4.BLM2: Paper Strips Divided Into Tenths*, and encourage students to use the strips when they work on the puzzles with their parents. (You may want to send home the puzzle solutions on *AddSub4.BLM3c* at the same time.) Make time in class for students to share their puzzle solutions and strategies.

LEARNING CONNECTION 1

Bedtime Treats!

MATERIALS

- base ten blocks (rods and small cubes) or paper strips cut from *AddSub4.BLM2: Paper Strips Divided Into Tenths*
- chocolate bar with ten sections*
- sheets of paper (1 per pair of students)

Describe the following scenario:

“Every night before going to bed, Lee’s grandparents have some chocolate. Because they are trying not to eat too much candy, they eat only part of a chocolate bar each time. The last time Lee visited his grandparents, he noticed that they had four chocolate bars in their “treat cupboard”.

Show students a chocolate bar divided into tenths. Ask:

- “How many equal sections does the chocolate bar have?”
- “How can you use base ten blocks or paper strips to represent all the chocolate bars Lee’s grandparents had in their cupboard?”

Have a student demonstrate how 4 rods or 4 paper strips could be used to represent the chocolate bars.

Explain that on the first night of Lee’s visit, his grandparents opened one chocolate bar and ate 6 sections. Ask the following questions:

- “What fraction of the bar did Lee’s grandparents eat?” (6 tenths)
- “What amount of the chocolate bars is left? (3 and 4 tenths) How do you know?”

Have students use base ten blocks or paper strips to explain the remaining amount of the chocolate bars. Together, discuss how to represent the situation using a number sentence. Record “4.0 – 0.6 = 3.4” on the board.

Record the following on the board:

- Night 2: 6 sections
- Night 3: 8 sections
- Night 4: 10 sections

* Ensure that the chocolate bar used is free of allergens.
Explain that Lee’s grandparents ate 6 sections on the second night of his visit, 8 sections on the third night, and 10 sections on the last night.

Have students work with a partner. Ask them to find the amount of chocolate that was left each night after Lee’s grandparents ate their treat. Encourage students to use base ten blocks, paper strips, and open number lines. Provide each pair of students with a sheet of paper on which they can record their work. Ask students to record a number sentence that represents the subtraction situation for each night.

Ask pairs of students to explain how they determined the remaining amount of chocolate each night. Discuss the meaning of the number sentences that students used to represent the subtraction situations.

**LEARNING CONNECTION 2**

**Closest to One**

**MATERIALS**
- spinners made from AddSub4.BLM4: Closest-to-One Spinner, a paper clip, and a pencil (1 per pair of students)
- base ten blocks (rods and small cubes)
- paper strips from AddSub4.BLM2: Paper Strip Divided Into Tenths
- sheets of paper (1 per student)

Explain the game to the class:
- Students play with a partner. Both players begin with a score of 10.
- One player spins the spinner, reads the number indicated by the spinner, and subtracts the number from 10. The player may use base ten blocks, paper strips divided into tenths, and/or open number lines to help him or her subtract. The player records the new score (the difference between 10 and the spinner number) on a piece of paper.
- The second player takes a turn.
- Players continue to take turns. On each turn, the player subtracts the number indicated on the spinner from the score achieved on his or her previous turn.
- Each player decides when he or she will stop spinning.
- The player with a final score that is closest to 1 wins the game.

Provide an opportunity for students to play a few rounds of the game. As students play the game, observe how they represent decimal amounts using concrete materials and decimal notation. Ask students to demonstrate how they use materials to help them subtract.
LEARNING CONNECTION 3
Decimal Number Triathlon

MATERIALS
• metre sticks
• paper and pencil
• AddSub4.BLM5: Decimal Number Triathlon (1 per group of 4 students)
• overhead transparency of AddSub4.BLM5: Decimal Number Triathlon
• overhead projector
• overhead marker

Create, or have your students suggest, three activities that meet the following requirements:
• The results of the activity can be measured to the nearest tenth of a metre.
• The activity can be done safely in the classroom (or in the gymnasium or on the playground).
• The activity requires minimal equipment.
• The activity is fun for all students.

Here are some examples of activities in which the results (distance) can be measured to the nearest tenth of a metre:
• flicking a penny with your thumb or finger;
• kicking a crumpled tissue or scrap of paper;
• blowing a cotton ball or ping-pong ball along a flat surface.

Organize students into groups of four. Have groups rotate through the three activities. (If possible, set up two centres for each event. Have half the class rotate through one set of three events, and the other half of the class through the other set.) At each centre, students perform the activity, measure one another’s results (to the closest tenth of a metre) using metre sticks, and record the results on a copy of AddSub4.BLM5: Decimal Number Triathlon.

After students have completed all three activities, reconvene the class. Invite individual students to share their results for the different activities, and use an overhead marker to fill in the chart on an overhead transparency of AddSub4.BLM5: Decimal Number Triathlon.

Pose questions that can be answered by adding or subtracting the data found in the chart (e.g., “What was the combined distance of Student A and Student B in the flick-a-penny event?”, “How much farther did Student C blow a cotton ball than Student D?”).

In groups of four, have students write three or four addition and subtraction questions that can be answered using the data in the chart. Ask students to work together as a group to answer the questions. Encourage students to use concrete materials and/or open number lines to help them find solutions.
LEARNING CONNECTION 4
Number-Line Bingo

MATERIALS
• spinners made from AddSub4.BLM4: Number-Line Bingo Spinner, a paper clip, and a pencil
  (1 per pair of students)
• sheets of paper (1 per student)
• AddSub4.BLM7: Number-Line Bingo (1 per pair of students)

Explain the game to the class:
• Students play the game with a partner.
• Players each spin the spinner twice and record the two numbers indicated by the spinner on
  a sheet of paper.
• Players use the numbers to create addition and subtraction sentences. For example, if a player’s
  numbers are 0.6 and 0.7, he or she could create the following addition and subtraction sentences:
  0.6 + 0.7 = 1.3; 0.7 + 0.6 = 1.3; or 0.7 – 0.6 = 0.1.
• Players exchange papers to check each other’s calculations.
• If players’ answers are correct, they circle the corresponding numbers (the sums or differences)
  on their individual number line on AddSub4.BLM7: Number-Line Bingo.
• Players continue to take turns spinning the spinner twice, creating addition and subtraction
  sentences, and circling answers on their number lines. (If a number is already circled, players
  do not need to circle the number again.)
• The first player to circle all the numbers on his or her number line, or the player who has the
  most numbers circled at the end of a specified time, wins the game.

eWORKSHOP CONNECTION
Visit www.eworkshop.on.ca for other instructional activities that focus on addition and subtraction
concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Addition and
Subtraction (4 to 6)”, and then click the number to the right of it.
What's the Capacity?

Dishwashing Detergent

Laundry Detergent

Water

Juice

Fabric Softener

1.4 L  0.5 L  3.6 L  2.9 L  1.1 L
Addition and Subtraction Puzzles

Dear Parent/Guardian:

We have been learning about adding and subtracting decimal numbers. Attached are some addition and subtraction puzzles for you and your child to solve together.

Have your child cut out the six circles containing the decimal numbers for the first puzzle. Ask your child to place the number circles on the empty circles in the triangle so that a true addition or subtraction sentence is formed on each side of the puzzle. Have your child glue the number circles onto the puzzle or write the numbers in the empty circles. Ask him or her to repeat for each of the remaining puzzles.

As you and your child work on these puzzles, encourage him or her to add and subtract decimal numbers using the paper strips divided into tenths. (You might look at the solution after finishing the puzzle, or use one of the correct numbers as a clue to help you get started.)

In class, students may want to share their solutions and strategies for these puzzles.

You and your child might try to create your own decimal number puzzle. If you do, have your child bring the puzzle to school for our class to solve.

Thank you for doing this activity with your child.
Addition and Subtraction Puzzles

1. \( + + = \) \( 0.1 \) \( 0.4 \) \( 0.5 \)
   \( - - = \) \( 0.7 \) \( 0.8 \) \( 1.2 \)

2. \( + + = \) \( 0.5 \) \( 0.6 \) \( 0.9 \)
   \( - - = \) \( 1.1 \) \( 1.4 \) \( 2.0 \)

3. \( + \) \( + \) \( = \) \( 0.4 \) \( 0.5 \) \( 0.6 \)
   \( - \) \( - \) \( = \) \( 0.7 \) \( 1.1 \) \( 1.3 \)
   \( - - = \) \( 1.4 \) \( 2.0 \)
Addition and Subtraction Puzzles (Solutions)

\[
\begin{align*}
0.4 & + 0.8 = 1.2 \\
0.1 & + 0.7 = 0.8 \\
0.5 & - 0.7 = 0.8
\end{align*}
\]

\[
\begin{align*}
0.6 & + 1.4 = 2.0 \\
0.5 & + 0.5 = 1.0 \\
1.1 & - 0.9 = 0.2
\end{align*}
\]

\[
\begin{align*}
0.7 + 1.4 = 2.1 \\
0.4 & + 1.4 = 1.8 \\
0.7 & - 0.5 = 0.2
\end{align*}
\]
Closest-to-One Spinner

Make a spinner using this page, a paper clip, and a pencil.
### Decimal Number Triathlon

<table>
<thead>
<tr>
<th>Students</th>
<th>Activity</th>
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Number-Line Bingo Spinner

Make a spinner using this page, a paper clip, and a pencil.
Number-Line Bingo

Player: __________________________

Player: __________________________
Grade 5 Learning Activity
Reaching a Goal

OVERVIEW
In this learning activity, students determine the amount of money that needs to be raised by students in junior grades in a school fundraising project. To solve this problem, students use a variety of addition and subtraction strategies, including paper-and-pencil and mental computational methods. After solving the problem, students discuss and reflect on the efficiency of various strategies.

BIG IDEAS
This learning activity focuses on the following big ideas:

Operational sense: Students explore a variety of strategies for adding and subtracting decimal numbers to hundredths (money amounts).

Relationships: Students compare decimal numbers expressed as money amounts. They also consider how number relationships help to determine appropriate and efficient computational strategies (e.g., when a mental strategy is more efficient than using a standard algorithm).

Representation: Students represent decimal numbers visually by using concrete materials, and symbolically by using decimal notation.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectation.
Students will:
• add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms (e.g., use 10 × 10 grids to add 2.45 and 3.25).

This specific expectation contributes to the development of the following overall expectation.
Students will:
• solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.
ABOUT THE LEARNING ACTIVITY

MATERIALS
• sheets of paper (1 per pair of students)
• a variety of concrete materials for representing decimal numbers to hundredths (e.g., money sets, 10 × 10 grids, base ten materials)
• half sheets of chart paper or large sheets of newsprint (1 per pair of students)
• markers (a few per pair of students)

MATH LANGUAGE
• decimal number • difference
• tenths • mental computation
• hundredths • algorithm
• sum

Note: Decimal numbers, such as 0.6 or 3.25, are often read as “point six” (or “decimal six”) and “three point two five”. To connect decimal numbers to their meaning, it is helpful to read 0.6 as “six tenths” and 3.25 as “three and twenty-five hundredths”.

INSTRUCTIONAL SEQUENCING
Before starting this learning activity, students should have had experiences with representing decimal numbers to hundredths using concrete materials, reading and writing money amounts to $1000, and adding and subtracting whole numbers using algorithms and mental strategies. This learning activity allows students to apply their understanding of decimal numbers (money amounts) and computational methods in solving a problem.

ABOUT THE MATH
By Grade 5, students have developed mental strategies and algorithms for adding and subtracting whole numbers. These whole-number strategies provide a foundation for learning computational methods with decimal numbers.

In this learning activity, students solve a problem that involves addition and subtraction of decimal numbers to hundredths (money amounts) by using strategies that make sense to them. The numbers in the problem have been chosen to allow for the use of mental strategies as well as algorithms. Students examine various approaches to solving the problem, reflect on the efficiency of various strategies, and discuss cases in which different strategies are appropriate.

GETTING STARTED
Explain the following problem:

“The Pine Hill School community hopes to raise $500 for a charity organization. The parent group held a craft sale and raised $265.50. The students in the primary grades raised $104.25 by selling tickets to a play they performed. Next week, the students...
in the junior grades will be holding a Skip-a-Thon to raise the rest of the money. How much money do the students in the junior grades need to raise?"

Discuss the problem with students, and record important information about the problem on the board:

- Fundraising goal: $500
- Parent group: $265.50
- Primary classes: $104.25
- Junior classes: $?

Pose the following questions. Have students explain their thinking.

- "About how much money has been raised so far?"
- "Do the students in the junior grades have to raise more money than the parent group did?"
- "Do the students in the junior grades have to raise more money than the students in the primary grades did?"
- "About how much money will the students in the junior grades need to raise to reach the goal?"

Explain that students will work with a partner to solve the problem.

WORKING ON IT

Provide each pair of students with a sheet of paper on which they can record their work. Encourage students to think about how they might use different strategies, including algorithms (paper-and-pencil calculations) and mental computations, to help them solve the problem. Explain that students must be able to explain their strategies later, when the class discusses different ways to solve the problem.

Note: Performing mental computations does not mean that students may not use paper and pencil. Often strategies involve both mental calculations and recording numbers on paper. Students may jot down numbers on paper to help them keep track of figures, but they should not perform paper-and-pencil calculations that can be done mentally.

Have available concrete materials for representing decimal numbers to hundredths (e.g., money sets, 10 × 10 grids, base ten materials) and explain to students that representing the problem by using the materials can help them solve the problem.

STRATEGIES STUDENTS MIGHT USE

FINDING THE DIFFERENCE BETWEEN $500 AND THE AMOUNT OF MONEY RAISED SO FAR

Students might add $265.50 and $104.25 to determine the amount of money raised so far ($369.75), and then calculate the difference between $369.75 and $500. Some students may use traditional addition and subtraction algorithms to perform these computations; however, other students may...
choose to use mental computations. To add $265.50 and $104.25, for example, they might use an adding-on strategy (see p. 21) by breaking $104.25 into parts, and adding on each part:

- adding $100 to $265.50 to get to $365.50;
- then adding $4 to $365.50 to get to $369.50;
- then adding $0.25 to $369.50 to get to $369.75.

This strategy can be modelled by using an open number line. (In an open number line, the jumps are not to scale.)

To calculate the difference between $369.75 and $500, students might use an adding-on strategy (see p. 21):

- add $0.25 to $369.75 to get to $370;
- then add $30 to $370 to get to $400;
- then add $100 to $400 to get to $500;
- then calculate the difference by adding $0.25 + $30 + $100. The difference is $130.25.

The following open number line illustrates this strategy.

SUBTRACTING THE AMOUNTS OF MONEY RAISED SO FAR FROM $500

Students might subtract $265.50 from $500 first, and then subtract $104.25 to determine the amount that remains to be raised. (Alternatively, they might subtract $104.25 first, and then subtract $265.50.) Students might decide to use a subtraction algorithm to perform these computations.

\[
\begin{array}{c}
499.00 \\
- 265.50 \\
\hline
233.50 \\
- 104.25 \\
\hline
130.25
\end{array}
\]

Note: Students may experience difficulty in using the standard algorithm, and may demonstrate little understanding of the regrouping required to perform the computations. If students are unable to explain the meaning of the algorithm, encourage them to find a method that they can understand and explain to others.
Students might also perform the subtraction computations in other ways. For example, they might combine parts of $265.50 and $104.25 according to place value, and subtract these parts from $500:

- combine the hundreds (i.e., $200 + $100 = $300), and subtract that amount from $500 (i.e., $500 – $300 = $200); then
- combine the tens (i.e., $60 + 0 = $60), and subtract that amount from $200 (i.e., $200 – $60 = $140); then
- combine the ones (i.e., $5 + $4 = $9), and subtract that amount from $140 (i.e., $140 – $9 = $131); then
- combine the hundredths (i.e., $0.50 + $0.25 = $0.75), and subtract that amount from $131 (i.e., $131 – $0.75 = $130.25).

Observe students as they are working. Ask them questions that allow them to explain and reflect on their strategies:

- “What strategy are you using to solve the problem?”
- “What did you do first? Why did you do that first? What did you do next?”
- “How are you using paper-and-pencil calculations? Mental computation?”
- “Which calculations did you do mentally? Why did you decide to do these calculations mentally?”
- “How could you show your strategy so that others can understand what you are thinking? How could you use a diagram, such as an open number line?”

It may be necessary to demonstrate how diagrams, such as open number lines, can be used to model strategies. (Examples of open number lines can be found on pages 17–18.)

After pairs of students have found a solution, provide them with markers and a half sheet of chart paper or a large sheet of newsprint. Ask students to record their strategies in a way that can be clearly understood by others. Remind students that they need to be prepared to explain their strategies to the class.

REFLECTING AND CONNECTING

Have pairs of students present their solutions to the class. Try to include a variety of computational strategies (e.g., various mental computation strategies, paper-and-pencil algorithms). Pose questions that encourage the presenters to explain the computational strategies they used to solve the problem:

- “How does your strategy work? What steps did you take to use this strategy? Why did you do that step?”
- “Why did you choose this strategy?”
- “What worked well with this strategy? What did not work well?”
- “Was this strategy easy to use? Why or why not?”
- “How do you know that your solution is correct?”
Assist the class in understanding the strategies that are presented. For example, have students explain a strategy in their own words to a partner. As well, model strategies by drawing diagrams (e.g., open number lines) on the board.

Post students’ work in the classroom following each presentation. After several pairs have explained their strategies, ask questions that help the class to reflect on the efficiency of different strategies:

- “Which strategies worked well in solving the problem? Why?”
- “How would you explain this strategy to someone who has never used it?”
- “When is it appropriate to use this strategy? For example, with what kinds of numbers does this strategy work well?”
- “Which strategy would you use if you solved another problem like this again? Why?”
- “How would you change a strategy? Why would you change it?”

Keep students’ work posted after the activity is finished. Students will need to use it during Learning Connection 1.

**ADAPTATIONS/EXTENSIONS**

Provide a simpler version of the problem for students who experience difficulties.

“The school community hopes to raise $500. If the school has already raised $295.50, how much more money does the school need to raise to reach its goal?”

Extend the activity for students requiring a greater challenge by asking them to find different ways to solve the problem. Ask students to judge the efficiency of the different strategies.

You might also challenge students by having them determine the amount of money that needs to be raised if the goal changes (e.g., from $500 to $750).

**ASSESSMENT**

Observe students to assess how well they:

- understand the problem and formulate an approach to solving it;
- select and apply appropriate computational strategies (e.g., mental strategies, algorithms);
- demonstrate flexibility and skill in using various computational strategies;
- represent and explain computational strategies (e.g., by using an open number line);
- judge the efficiency of different computational strategies.

**HOME CONNECTION**

Send home copies of AddSub5.BLM1: Adding and Subtracting Money Amounts. In this Home Connection activity, students demonstrate different strategies for adding and subtracting money amounts.
LEARNING CONNECTION 1
How Can We Add?

MATERIALS
• spinners made from AddSub.BLM2: 0–9 Spinner, a paper clip, and a pencil (1 per pair of students)
• sheets of paper (a few per pair of students)
• students’ posted work samples from the main learning activity (half sheets of chart paper or large sheets of newsprint showing computational strategies)

Provide each pair of students with the materials they need to make a spinner (a copy of AddSub.BLM2: 0–9 Spinner, a pencil, and a paper clip). Instruct pairs to draw the following “blank” number expression on a sheet of paper:

__ . ____ ____ + ____ . ____ ____

Next, have partners take turns spinning the spinner and recording the digit it shows in one of the six spaces of the number expression. Once a digit has been recorded, it cannot be erased and recorded in another position.

After all spaces are filled, have students discuss how they can add the two decimal numbers mentally. Encourage them to refer to the posted work samples from the main learning activity to review different mental strategies and to select an appropriate method. Remind students that they may jot down numbers on paper to help them keep track of figures, but that they should not perform paper-and-pencil calculations that can be done mentally.

After pairs of students calculate the sum of their decimal numbers, have them record an explanation of the steps they took to add the numbers mentally. Encourage them to use diagrams, such as open number lines, to show their thinking.

Combine pairs of students to create groups of four. Have pairs explain to their new partners how they calculated the sum of the decimal numbers mentally.

Adaptation: Students could also spin the spinner six times and record the digits on a piece of paper. After obtaining six digits from the spinner, they record the digits in the blank number expression so as to determine the greatest possible sum (or the sum that is closest to 10).

LEARNING CONNECTION 2
What Is the Question?

MATERIALS
• AddSub.BLM3: What Is the Question? (1 per pair of students)

Provide pairs of students with a copy of AddSub.BLM3: What Is the Question? Have students place decimal points in the numbers being added or subtracted so that each equation is correct. Discuss the methods that students used to determine solutions.
LEARNING CONNECTION 3
Problem Solving at the Fair

MATERIALS
• AddSub5.BLM4a-b: Problem Solving at the Fair (1 per pair of student)
• half sheets of chart paper or large sheets of newsprint (1 sheet per pair of students)
• markers (a few per pair of students)

Have pairs of students work together to solve the problems on AddSub5.BLM4a-b: Problem Solving at the Fair using the information given in the table on AddSub5.BLM4a: Problem Solving at the Fair. Provide each pair with markers and a half sheet of chart paper or a large sheet of newsprint on which they can record their strategies and solutions.

Ask students to present their strategies and solutions to the class. Ask questions such as:
• “Why did you use this strategy?”
• “What worked well with this strategy? What did not work well?”
• “How do you know that your solution is correct?”

Have students compare the different strategies.

eWORKSHOP CONNECTION
Visit www.eworkshop.on.ca for other instructional activities that focus on addition and subtraction concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Addition and Subtraction (4 to 6)”, and then click the number to the right of it.
Adding and Subtracting Money Amounts

Dear Parent/Guardian:

We have been learning to add and subtract money amounts. Sometimes it is easy to calculate answers “in your head”. At other times, it is easier to figure out answers using paper and pencil.

Provide an opportunity for your child to explain different ways to add and subtract money amounts:

• Together, choose two items in a catalogue or grocery flyer that each cost between $5 and $10.
• Have your child determine the total cost of the two items. Ask your child to explain how he or she added the numbers.
• Ask your child to show a different way to determine the total cost.
• Have your child figure out how much money he or she would get back if he or she used $20 to purchase the two items. Again, ask your child to determine a different way to solve the problem.
• You might show your child how you would add and subtract the money amounts.

Thank you for providing an opportunity for your child to show different ways to add and subtract money amounts.
0–9 Spinner

Make a spinner using this page, a paper clip, and a pencil.
What Is the Question?

64 + 325 = 3.89

48 + 327 + 122 = 20.27

272 + 143 + 135 = 42.13

1821 + 74 + 109 + 24 = 29.1

2367 – 211 = 21.56

4974 – 366 = 493.74

321 – 158 + 6739 = 69.02
Problem Solving at the Fair

The fair is about to get started, and vendors are busy setting up their booths. The following table shows how much each of the vendors paid for a booth, and what they paid for their products. It also shows the amount of money they collected during the three-day fair.

Use the information in the table to solve the problems on the next page.

<table>
<thead>
<tr>
<th>Booth</th>
<th>Cost of Booth</th>
<th>Cost of Products</th>
<th>Money Collected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom’s Hand-Picked Apples</td>
<td>$25 for all 3 days</td>
<td>$15 (for apples)</td>
<td>$87.35</td>
</tr>
<tr>
<td>Marlena’s Clay Creations</td>
<td>$8.75 per day</td>
<td>$25 (for clay and fuel for the kiln)</td>
<td>$173.85</td>
</tr>
<tr>
<td>Bjorn’s “Bottle Blast” Game</td>
<td>$10.50 per day</td>
<td>$66.50 (for prizes)</td>
<td>$198</td>
</tr>
<tr>
<td>Rhoda’s Refreshment Booth</td>
<td>$21 for all 3 days</td>
<td>$58.75 (for juice mix and supplies)</td>
<td>$163.50</td>
</tr>
</tbody>
</table>
| McKendrick Family’s Handmade Scarves and Mitts | $7.50 per day | $35 (for wool) | $156.30
Problem Solving at the Fair

Use information from the table on the previous page to solve the following problems. Explain how you calculated your answers.

1. Which vendor paid the least amount of money for a booth?

2. Which vendor made the most money (profit) at the end of the fair?

3. Which vendor made the least amount of money?

4. Would the vendor who made the most money at the fair still have made the most if the cost of the booths had been the same for everyone?
Grade 6 Learning Activity
A Weighty Matter

OVERVIEW
In this learning activity, students review the meaning of “thousandths” by representing decimal numbers using base ten materials. After discussing concerns about heavy backpacks and possible related injuries, students find combinations of backpack items whose total mass is close to, but does not exceed, the recommended maximum mass.

BIG IDEAS
This learning activity focuses on the following big ideas:

**Quantity:** Using base ten materials and mass sets, students explore the “howmuchness” of decimal numbers to thousandths.

**Operational sense:** Students explore a variety of strategies for adding decimal numbers.

**Relationships:** Representing decimal numbers using base ten materials allows students to understand the base ten relationships in our number system (e.g., 10 thousandths is 1 hundredth).

**Representation:** Students represent decimal numbers visually by using concrete materials, and symbolically by using decimal notation.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations. Students will:

- represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);
- demonstrate an understanding of place value in whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools and strategies (e.g., use base ten materials to represent the relationship between 1, 0.1, 0.01, and 0.001);
- add and subtract decimal numbers to thousandths, using concrete materials, estimation, algorithms, and calculators.

These specific expectations contribute to the development of the following overall expectation. Students will:

- solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies.
ABOUT THE LEARNING ACTIVITY

MATERIALS
• half sheets of chart paper or large sheets of newsprint (1 per group of 4 students)
• markers (a few per group of 4 students)
• base ten blocks (large cubes, flats, rods, small cubes)
• kilogram and gram masses (1 kg mass and 1 g mass per group of 4 students)
• bathroom scale (optional)
• AddSub6.BLM1: Backpack items (1 per pair of students)
• sheets of paper (a few per pair of students)
• sheets of chart paper or large sheets of newsprint (1 per pair of students)
• AddSub6.BLM2: Lessen the Load (1 per pair of students)

MATH LANGUAGE
• decimal number
• tenths
• hundredths
• thousandths
• whole
• place value
• kilogram
• gram

Note: Decimal numbers, such as 0.61 or 3.254, are often read as “point six one” (or “decimal six one”) and “three point two five four”. To connect decimal numbers to their meaning, it is helpful to read 0.61 as “sixty-one hundredths” and 3.254 as “three and two hundred fifty-four thousandths”.

INSTRUCTIONAL SEQUENCING
This learning activity reviews the meaning of thousandths and provides an opportunity for students to explore strategies for adding decimal numbers. Before starting this learning activity, students should have had experiences with representing decimal numbers to hundredths using concrete materials, recording hundredths using decimal notation, and adding decimal numbers to hundredths.

ABOUT THE MATH
In Grade 6, students continue to build on their understanding of decimal numbers by exploring the meaning of thousandths. Opportunities to represent decimal numbers using concrete materials (e.g., base ten blocks, place-value mats) help students to develop an understanding of the base ten relationships in decimal numbers.

Grade 6 students also learn to add and subtract decimal numbers to thousandths. In this learning activity, students have an opportunity to add decimal numbers using a variety of methods (e.g., using manipulatives, using student-generated strategies, using algorithms). The learning activity also provides experience in estimating the sums of decimal numbers.
GETTING STARTED

The following two activities reinforce an understanding of “thousandths” and prepare students for the problem in the main learning activity.

ACTIVITY 1: BASE TEN INVESTIGATION

Organize students into groups of four. Provide each group with markers and a half sheet of chart paper or a large sheet of newsprint. Have them create a place-value mat by vertically folding the paper into fourths, outlining the columns with a marker, and labelling the columns “Ones”, “Tenths”, “Hundredths”, “Thousandths”.

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
</table>

Provide each group with a collection of base ten blocks (large cubes, flats, rods, small cubes).

Record “1.3, 1.42, 2.09” on the board, and have students read the numbers orally (“one and three tenths”, “one and forty-two hundredths”, “two and nine hundredths”).

Tell students that the flat represents one whole. Ask them to work together as a group to represent the numbers recorded on the board using base ten blocks and their place-value mats. Ask students to demonstrate and explain how they used the materials to represent each number.

Next, explain that the large cube now represents one whole, and again have students represent the numbers recorded on the board. Discuss students’ representations with the base ten blocks, and how the change in the representation of one whole (i.e., the large cube instead of the flat) affected the concrete representation of the decimal numbers.

Remind students that the large cube still represents one whole. Show the class the following collections of base ten blocks:

- 1 large cube, 2 flats, 7 rods
- 2 large cubes, 6 rods

For each collection, have students identify the decimal number orally, and ask them to record the number using decimal notation. For example, 1 large cube, 2 flats, 7 rods represents one and twenty-seven hundredths, which can be recorded as “1.27”.

Provide each group with a collection of base ten blocks (large cubes, flats, rods, small cubes).
Next, show a small cube, and ask students to discuss in their groups the value of the small cube if the large cube has a value of one. Discuss the idea that the small cube is one thousandth of the large cube, and that “one thousandth” can be recorded as “1/1000” and “0.001”.

Show the following collections of base ten blocks, and ask students to identify and record each decimal number:

- 3 flats, 1 small cube ("three hundred one thousandths", 0.301)
- 1 large cube, 9 small cubes ("one and nine thousandths", 1.009)
- 2 large cubes, 3 rods, 1 small cube ("two and thirty-one thousandths", 2.031)

**ACTIVITY 2: BACKPACK INVESTIGATION**

Review the relationship between kilogram and gram. Provide each group of four students with a kilogram mass and a gram mass. Pose the following question: “If you place a kilogram mass in the ones column on the place-value mat, where would you place the gram mass?” Have students, in their groups, discuss their ideas. Then, review the concept that a gram is one thousandth of a kilogram (1/1000 or 0.001 of a kilogram).

Record the following on the board:

- 6 g = _______ kg
- 78 g = ________ kg
- 354 g = ________ kg

Have students work together in their groups to determine the missing decimal numbers in each number sentence. Discuss the solutions as a whole class.

Talk to the class about the health concerns related to the mass of students’ backpacks – students who carry heavy backpacks risk back, shoulder, and neck injuries. Explain that studies have found that the maximum mass a backpack should be is 15% of the student’s mass – about 5 kg for most Grade 6 students. Explain that a recent survey found that there was a wide range in the mass of students’ backpacks, but that many students were carrying backpacks that exceeded the recommended maximum mass.

You might ask a few students to show their backpacks to the class, and have the class predict if the backpack has a mass of more than 5 kg. Use a bathroom scale to check the predictions.

**WORKING ON IT**

Arrange students in pairs. Provide each pair with a copy of AddSUB6.BLM1: Backpack Items and a few sheets of blank paper. Provide access to base ten blocks, place-value mats, and mass sets, and encourage students to use the materials, if needed.

Explain that students will work with their partner to find combinations of items that come close to, but do not exceed, the recommended maximum mass of a backpack (5 kg). Encourage students to record combinations of different items and their total mass on the paper provided. Ask students to find more than one combination of items, challenging them to get as close to 5 kg as possible without going over.
As students work, observe their strategies, and ask the following questions:

- “How are you solving the problem?”
- “What strategy are you using to determine the total mass of the items?”
- “Can you calculate the total using another strategy?”
- “How are you using manipulatives/mass sets/drawings/computation to help you determine the total mass?”
- “Which numbers were easy to add? Why?”
- “Which combination of items is closest to 5 kg? How do you know?”

After students have had an opportunity to find different combinations of items, have them select the combination whose mass is closest to 5 kg. Provide each pair of students with markers and a sheet of chart paper or a large sheet of newsprint. Ask them to record the combination of items and the total mass. Have students record their strategies for adding the numbers in such a way that others will understand their thinking.

REFLECTING AND CONNECTING

Have pairs of students present their solutions and strategies to the class. Try to include two pairs who used different strategies (e.g., concrete materials, student-generated strategies, standard algorithms). Make positive comments about students’ work, being careful not to infer that some approaches are better than others. Your goal is to have students determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Post students’ work, and ask questions such as:

- “Which strategies are similar? How are they alike?”
- “Which strategy would you use if you solved a problem like this again?”
- “How would you change any of the strategies that were presented? Why?”
- “Which work clearly explains a solution? Why is the work clear and easy to understand?”

Discuss how determining the exact mass of backpack items is not practical in a real-life situation (although it provided a context for a math activity), and that estimating the mass of combinations of items might be a more appropriate approach.

Ask students to estimate the combined mass of two or three items on AddSub.BLM1: Backpack Items (e.g., gym clothes and shoes; math textbook, binder, and agenda). Discuss students’ estimation strategies. For example, to estimate the mass of a math textbook, a binder, and an agenda, students might recognize that the combined mass of the binder and agenda would be approximately 1 kg, and that the math textbook has a mass of approximately 1.5 kg. The estimated mass of the three items would be approximately 2.5 kg.

ADAPTATIONS/EXTENSIONS

Pair students who might have difficulty with a partner who can help them understand the problem and different strategies, including the use of concrete materials (e.g., base ten blocks, place-value mats).
To simplify the activity for students experiencing difficulties, provide a list of fewer backpack items with masses given as tenths of a kilogram (e.g., math textbook: 1.4 kg).

Challenge students by asking them to determine the mass of an actual filled backpack using a scale and mass sets. Students could determine whether the mass of the filled backpack is less than 5 kg.

Ask students who understand percents to calculate 15% of their own mass and to determine whether the mass of their filled backpack is acceptable (less than 15% of their personal mass).

**ASSESSMENT**

Observe students to assess how well they:

- represent decimal numbers using materials (e.g., base ten blocks, place-value mats);
- read and record decimal numbers;
- explain concepts related to place value (e.g., 10 thousandths are 1 hundredth);
- use appropriate strategies to add decimal numbers;
- explain their strategies for adding decimal numbers;
- use appropriate estimation strategies.

**HOME CONNECTION**

Send home copies of AddSub6.BLM2: Lessen the Load. This Home Connection activity suggests that parents and students examine the contents of the student’s backpack and remove any unneeded items. Parents and students are also encouraged to calculate the total mass of the student’s backpack.

Before AddSub6.BLM2: Lessen the Load is sent home, students could fill in the blank spaces in the chart with the names and masses of other backpack items.

**LEARNING CONNECTION 1**

**Weighty Names**

**MATERIALS**

- AddSub6.BLM3: Weighty Names (1 per pair of students)

Ask students if they have ever thought about how “heavy” their name is. Tell them they will have a chance to figure out the “mass” of their name. Give each student a copy of AddSub6.BLM3: Weighty Names, and discuss the example. If necessary, do another example with the class. Allow students to work with a partner to determine the mass of their names. (Students can determine the mass of their first and last names.)

Allow students to share the masses of their names. Have students determine whose name is the heaviest, lightest, and closest to 1 kg. Challenge students to find a name that has a mass of exactly 1 kg.
Students could take home a copy of AddSub6.BLM3: Weighty Names and determine the mass of family members’ names.

LEARNING CONNECTION 2
Missing the Point

Record the following number sentences on the board:
• $7.39 + 24.267 = 31\text{6}57$
• $25.398 + 9822 + 3.05 = 3827$
• $41562 - 33.257 = 8.305$
• $40.03 - 1967 = 20.36$

Explain that the decimal point is missing from one number in each number sentence. Have students work with a partner to copy the number sentences and to insert each missing decimal point. Explain to students that they are to use estimation, rather than computation with a paper and pencil or a calculator.

As a class, discuss strategies for determining the correct placement of the decimal points.

LEARNING CONNECTION 3
Surmising Sums

Record the following numbers on the board:
1.962 2.247 2.228 0.772
2.431 0.038 2.569 1.753

Have students work with a partner to determine which two decimal numbers have a sum of 2. Encourage students to use inspection, rather than paper-and-pencil calculations, to find the two numbers. Have students explain the strategies they used.

Repeat by having pairs of students find the two decimal numbers that have a sum of 3, 4, and 5. Have students explain their strategies.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on addition and subtraction concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Addition and Subtraction (4 to 6)”, and then click the number to the right of it.
Backpack Items

What combination of items could you put in a backpack so that the mass of the backpack comes close to, but does not exceed, the recommended maximum mass of 5 kg? You may include items more than once. Include the mass of the backpack (1.275 kg) in your total.

Find different combinations of items.

<table>
<thead>
<tr>
<th>Backpack Item</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>math textbook</td>
<td>1.395 kg</td>
</tr>
<tr>
<td>binder</td>
<td>0.764 kg</td>
</tr>
<tr>
<td>workbook</td>
<td>0.102 kg</td>
</tr>
<tr>
<td>agenda</td>
<td>0.245 kg</td>
</tr>
<tr>
<td>paperback novel</td>
<td>0.140 kg</td>
</tr>
<tr>
<td>pencil</td>
<td>0.005 kg</td>
</tr>
<tr>
<td>calculator</td>
<td>0.075 kg</td>
</tr>
<tr>
<td>gym clothes</td>
<td>0.485 kg</td>
</tr>
<tr>
<td>shoes</td>
<td>0.598 kg</td>
</tr>
<tr>
<td>lunch</td>
<td>0.582 kg</td>
</tr>
<tr>
<td>pencil case and contents</td>
<td>0.302 kg</td>
</tr>
<tr>
<td>geometry set</td>
<td>0.109 kg</td>
</tr>
</tbody>
</table>
Lessen the Load

Dear Parent/Guardian:

Our class has been investigating the problem of overweight backpacks. Studies have shown that students should carry no more than 15 percent of their body weight or mass (about 5 kilograms for most Grade 6 students) to prevent injury to back, neck, and shoulders. We conducted an investigation in which we found the total mass of various backpack items. The activity provided an opportunity for students to add decimal numbers.

Find some time for you and your child to go through your child’s backpack and to remove any unnecessary items. To reinforce an understanding about addition of decimal numbers, you and your child could use the following chart to find the total mass of your child’s filled backpack.

<table>
<thead>
<tr>
<th>Backpack Item</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>math textbook</td>
<td>1.395 kg</td>
</tr>
<tr>
<td>binder</td>
<td>0.764 kg</td>
</tr>
<tr>
<td>workbook</td>
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</tr>
<tr>
<td>agenda</td>
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</tr>
<tr>
<td>paperback novel</td>
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</tr>
<tr>
<td>pencil</td>
<td>0.005 kg</td>
</tr>
<tr>
<td>calculator</td>
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</tr>
<tr>
<td>gym clothes</td>
<td>0.485 kg</td>
</tr>
<tr>
<td>shoes</td>
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</tr>
<tr>
<td>lunch</td>
<td>0.582 kg</td>
</tr>
<tr>
<td>pencil case and contents</td>
<td>0.302 kg</td>
</tr>
<tr>
<td>geometry set</td>
<td>0.109 kg</td>
</tr>
</tbody>
</table>

Thank you for doing this activity with your child.
Weighty Names

How "heavy" is your name? Use the following key to determine the "mass" of your name.

Example: Alison = A + L + I + S + O + N

= 0.001 + 0.144 + 0.081 + 0.361 + 0.225 + 0.196

= 1.008 kg, or 1 kg and 8 g

MASS OF LETTERS IN KILOGRAMS

<table>
<thead>
<tr>
<th>Letter</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.001</td>
</tr>
<tr>
<td>B</td>
<td>0.004</td>
</tr>
<tr>
<td>C</td>
<td>0.009</td>
</tr>
<tr>
<td>D</td>
<td>0.016</td>
</tr>
<tr>
<td>E</td>
<td>0.025</td>
</tr>
<tr>
<td>F</td>
<td>0.036</td>
</tr>
<tr>
<td>G</td>
<td>0.049</td>
</tr>
<tr>
<td>H</td>
<td>0.064</td>
</tr>
<tr>
<td>I</td>
<td>0.081</td>
</tr>
<tr>
<td>J</td>
<td>0.100</td>
</tr>
<tr>
<td>K</td>
<td>0.121</td>
</tr>
<tr>
<td>L</td>
<td>0.144</td>
</tr>
<tr>
<td>M</td>
<td>0.169</td>
</tr>
<tr>
<td>N</td>
<td>0.196</td>
</tr>
<tr>
<td>O</td>
<td>0.225</td>
</tr>
<tr>
<td>P</td>
<td>0.256</td>
</tr>
<tr>
<td>Q</td>
<td>0.289</td>
</tr>
<tr>
<td>R</td>
<td>0.324</td>
</tr>
<tr>
<td>S</td>
<td>0.361</td>
</tr>
<tr>
<td>T</td>
<td>0.400</td>
</tr>
<tr>
<td>U</td>
<td>0.441</td>
</tr>
<tr>
<td>V</td>
<td>0.484</td>
</tr>
<tr>
<td>W</td>
<td>0.529</td>
</tr>
<tr>
<td>X</td>
<td>0.576</td>
</tr>
<tr>
<td>Y</td>
<td>0.625</td>
</tr>
<tr>
<td>Z</td>
<td>0.676</td>
</tr>
</tbody>
</table>
Number Sense and Numeration, Grades 4 to 6

Volume 3

Multiplication

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

Ontario Education excellence for all

2006
Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.
Number Sense and Numeration, Grades 4 to 6

Volume 3
Multiplication

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6
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INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of The Ontario Curriculum, Grades 1–8: Mathematics, 2005. This guide provides teachers with practical applications of the principles and theories that are elaborated in A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006.

The guide comprises the following volumes:
- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

The present volume – Volume 3: Multiplication – provides:
- a discussion of mathematical models and instructional strategies that support student understanding of multiplication;
- sample learning activities dealing with multiplication for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume also contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by “eLearning modules” that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pp. 43, 57, and 69).
Relating Mathematics Topics to the Big Ideas

The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the “big ideas” of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005, p. 4)

In planning mathematics instruction, teachers generally develop learning opportunities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

• quantity
• operational sense
• proportional reasoning
• representation
• relationships

Each of the big ideas is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a learning activity about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

• problem solving
• reasoning and proving
• reflecting
• selecting tools and computational strategies
• connecting
• representing
• communicating
The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

**Problem Solving:** Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

**Reasoning and Proving:** The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions that teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

**Reflecting:** Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

**Selecting Tools and Computational Strategies:** Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.
Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students’ own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners. The following table outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.
## Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
</table>
| **Intellectual development** | Generally, students in the junior grades:  
• prefer active learning experiences that allow them to interact with their peers;  
• are curious about the world around them;  
• are at a concrete operational stage of development, and are often not ready to think abstractly;  
• enjoy and understand the subtleties of humour. | The mathematics program should provide:  
• learning experiences that allow students to actively explore and construct mathematical ideas;  
• learning situations that involve the use of concrete materials;  
• opportunities for students to see that mathematics is practical and important in their daily lives;  
• enjoyable activities that stimulate curiosity and interest;  
• tasks that challenge students to reason and think deeply about mathematical ideas. |
| **Physical development** | Generally, students in the junior grades:  
• experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys);  
• are concerned about body image;  
• are active and energetic;  
• display wide variations in physical development and maturity. | The mathematics program should provide:  
• opportunities for physical movement and hands-on learning;  
• a classroom that is safe and physically appealing. |
| **Psychological development** | Generally, students in the junior grades:  
• are less reliant on praise but still respond well to positive feedback;  
• accept greater responsibility for their actions and work;  
• are influenced by their peer groups. | The mathematics program should provide:  
• ongoing feedback on students’ learning and progress;  
• an environment in which students can take risks without fear of ridicule;  
• opportunities for students to accept responsibility for their work;  
• a classroom climate that supports diversity and encourages all members to work cooperatively. |
| **Social development** | Generally, students in the junior grades:  
• are less egocentric, yet require individual attention;  
• can be volatile and changeable in regard to friendship, yet want to be part of a social group;  
• can be talkative;  
• are more tentative and unsure of themselves;  
• mature socially at different rates. | The mathematics program should provide:  
• opportunities to work with others in a variety of groupings (pairs, small groups, large group);  
• opportunities to discuss mathematical ideas;  
• clear expectations of what is acceptable social behaviour;  
• learning activities that involve all students regardless of ability. |

(continued)
### Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moral and ethical development</td>
<td>Generally, students in the junior grades: • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask “why” questions; • begin to consider others’ points of view.</td>
<td>The mathematics program should provide: • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas and evaluate the ideas of others.</td>
</tr>
</tbody>
</table>

(Adapted, with permission, from *Making Math Happen in the Junior Grades*. Elementary Teachers’ Federation of Ontario, 2004.)
LEARNING ABOUT MULTIPLICATION IN THE JUNIOR GRADES

Introduction
The development of multiplication concepts represents a significant growth in students’ mathematical thinking. With an understanding of multiplication, students recognize how groups of equal size can be combined to form a whole quantity. Developing a strong understanding of multiplication concepts in the junior grades builds a foundation for comprehending division concepts, proportional reasoning, and algebraic thinking.

PRIOR LEARNING
In the primary grades, students explore the meaning of multiplication by combining groups of equal size. Initially, students count objects one by one to determine the product in a multiplication situation. For example, students might use interlocking cubes to represent a problem involving four groups of three, and then count each cube to determine the total number of cubes.

With experience, students learn to use more sophisticated counting and reasoning strategies, such as using skip counting and using known addition facts (e.g., for 3 groups of 6: 6 plus 6 is 12, and 6 more is 18). Later, students develop strategies for learning basic multiplication facts, and use these facts to perform multiplication computations efficiently.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES
In the junior grades, instruction should focus on developing students’ understanding of multiplication concepts and meaningful computational strategies, rather than on having students memorize the steps in algorithms. Learning experiences need to contribute to students’
understanding of part-whole relationships – that is, groups of equal size (the parts) can be combined to create a new quantity (the whole).

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to multiplication, listed in the following table.

<table>
<thead>
<tr>
<th>Curriculum Expectations Related to Multiplication, Grades 4, 5, and 6</th>
<th>By the end of Grade 4, students will:</th>
<th>By the end of Grade 5, students will:</th>
<th>By the end of Grade 6, students will:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Expectations</td>
<td>• solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies;</td>
<td>• solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies;</td>
<td>• solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;</td>
</tr>
<tr>
<td>Specific Expectations</td>
<td>• multiply to 9 × 9 and divide to 81÷ 9, using a variety of mental strategies;</td>
<td>• multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms;</td>
<td>• use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution;</td>
</tr>
<tr>
<td></td>
<td>• solve problems involving the multiplication of one-digit whole numbers, using a variety of mental strategies;</td>
<td>• multiply decimal numbers by 10, 100, 1000, and 10 000, and divide decimal numbers by 10 and 100, using mental strategies;</td>
<td>• describe multiplicative relationships between quantities by using simple fractions and decimals;</td>
</tr>
<tr>
<td></td>
<td>• multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100, using mental strategies;</td>
<td>• use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution;</td>
<td>• demonstrate an understanding of simple multiplicative relationships involving whole-number rates, through investigation using concrete materials and drawings.</td>
</tr>
</tbody>
</table>

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)
The following sections explain content knowledge related to multiplication concepts in the junior grades, and provide instructional strategies that help students develop an understanding of multiplication. Teachers can facilitate this understanding by helping students to:

• interpret multiplication situations;
• use models to represent multiplication;
• learn basic multiplication facts;
• develop skills in multiplying by multiples of 10;
• develop a variety of computational strategies;
• develop strategies for multiplying decimal numbers;
• develop effective estimation strategies for multiplication;
• relate multiplication and division.

Interpreting Multiplication Situations

Solving a variety of multiplication problems helps students to understand how the operation can be applied in different situations. Types of multiplication problems include equal-group problems and multiplicative-comparison problems.

Equal-group problems involve combining sets of equal size. 

Examples:
• In a classroom, each work basket contains 5 markers. If there are 6 work baskets, how many markers are there?
• How many eggs are there in 3 dozen?
• Kendra bought 4 packs of stickers. Each pack cost $1.19. How much did she pay?

Multiplicative-comparison problems involve a comparison between two quantities in which one is described as a multiple of the other. In multiplicative-comparison problems, students must understand expressions such as “3 times as many”. This type of problem helps students to develop their ability to reason proportionally.

Examples:
• Luke’s dad is four times older than Luke is. If Luke is 9 years old, how old is his dad?
• Last Tuesday there was 15 cm of snow on the ground. The amount of snow has tripled since then. About how much snow is on the ground now?
• Felipe’s older sister is trying to save money. This month she saved 5 times as much money as she did last month. Last month she saved $5.70. How much did she save this month?

Students require experiences in interpreting both types of problems and in applying appropriate problem-solving strategies. It is not necessary, though, that students be able to identify or define these problem types.
Using Models to Represent Multiplication

Models are concrete and pictorial representations of mathematical ideas. It is important that students have opportunities to represent multiplication using materials such as counters, interlocking cubes, and base ten blocks. For example, students might use base ten blocks to represent a problem involving $4 \times 24$.

By regrouping the materials into tens and ones (and trading 10 ones cubes for a tens rod), students determine the total number of items.

Students can also model multiplication situations on number lines. Jumps of equal length on a number line reflect skip counting – a strategy that students use in early stages of multiplying. For example, a number line might be used to compute $4 \times 3$.

Later, students can use open number lines (number lines on which only significant numbers are indicated) to show multiplication with larger numbers. The following number line shows $4 \times 14$. 

14 Number Sense and Numeration, Grades 4 to 6 – Volume 3
An array (an arrangement of objects in rows and columns) provides a useful model for multiplication. In an array, the number of items in each row represents one of the factors in the multiplication expression, while the number of columns represents the other factor. Consider the following problem.

"Amy’s uncle has a large stamp collection. Her uncle displayed all his stamps from Australia on a large sheet of paper. Amy noticed that there were 8 rows of stamps with 12 stamps in each row. How many Australian stamps are there?"

To solve this problem, students might arrange square tiles in an array, and use various strategies to determine the number of tiles. For example, they might count the tiles individually, skip count groups of tiles, add 8 twelve times, or add 12 eight times. Students might also observe that the array can be split into two parts: an $8 \times 10$ part and an $8 \times 2$ part. In doing so, they decompose $8 \times 12$ into two multiplication expressions that are easier to solve, and then add the partial products to determine the product for $8 \times 12$.

After students have had experiences with representing multiplication using arrays (e.g., making concrete arrays using tiles; drawing pictorial arrays on graph paper), teachers can introduce open arrays as a model for multiplication. In an open array, the squares or individual objects are not indicated within the interior of the array rectangle; however, the factors of the multiplication expression are recorded on the length and width of the rectangle. An open array does not have to be drawn to scale. Consider this problem.

"Eli helped his aunt make 12 bracelets for a craft sale. They strung 14 beads together to make each bracelet. How many beads did they use?"

The open array may not represent how students visualize the problem (i.e., the groupings of beads), nor does it provide an apparent solution to $12 \times 14$. The open array does, however, provide a tool with which students can reason their way to a solution. Students might realize that
10 bracelets of 14 beads would include 140 beads, and that the other two bracelets would include 28 beads \((2 \times 14 = 28)\). By adding 140 + 28, students are able to determine the product of \(12 \times 14\).

The splitting of an array into parts (e.g., dividing a \(12 \times 14\) array into two parts: \(10 \times 14\) and \(2 \times 14\)) is an application of the distributive property. The property allows a factor in a multiplication expression to be decomposed into two or more numbers, and those numbers can be multiplied by the other factor in the multiplication expression.

Initially, mathematical models, such as open arrays, are used by students to represent problem situations and their mathematical thinking. With experience, students can also learn to use models as powerful tools with which to think (Fosnot & Dolk, 2001). Appendix 3–1: Using Mathematical Models to Represent Multiplication provides guidance to teachers on how they can help students use models as representations of mathematical situations, as representations of mathematical thinking, and as tools for learning.

**Learning Basic Multiplication Facts**

A knowledge of basic multiplication facts supports students in understanding multiplication concepts, and in carrying out more complex computations with multidigit multiplication. Students who do not have quick recall of facts often get bogged down and become frustrated when solving a problem. It is important to note that recall of multiplication facts does not necessarily indicate an understanding of multiplication concepts. For example, a student may have memorized the fact \(5 \times 6 = 30\) but cannot create their own multiplication problem requiring the multiplication of five times six.

The use of models and thinking strategies helps students to develop knowledge of basic facts in a meaningful way. Chapter 10 in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006* (Volume 5) provides practical ideas on ways to help students learn basic multiplication facts.

**Developing Skills in Multiplying by Multiples of 10**

Because many strategies for multidigit multiplication depend on decomposing numbers to hundreds, tens, and ones, it is important that students develop skill in multiplying numbers by multiples of 10. For example, students in the junior grades should recognize patterns such as \(7 \times 8 = 56\), \(7 \times 80 = 560\), \(7 \times 800 = 5600\), and \(7 \times 8000 = 56\,000\).

Students can use models to develop an understanding of why patterns emerge when multiplying by multiples of 10. Consider the relationship between \(3 \times 2\) and \(3 \times 20\).
An array can be used to show $3 \times 2$:

```
2
3

3 \times 2 = 6
```

By arranging ten $3 \times 2$ arrays in a row, $3 \times 20$ can be modelled using an array. The array shows that $3 \times 20$ is also 10 groups of 6, or 60.

```
20
3

3 \times 20 = 60
```

Students can also use base ten materials to model the effects of multiplying by multiples of 10. The following example illustrates $3 \times 2$, $3 \times 20$, and $3 \times 200$.

Three rows of 2 ones cubes:

```
3 \times 2 = 6
```

Three rows of 2 tens rods:

```
3 \times 20 = 60
```

Three rows of 2 hundreds flats:

```
3 \times 200 = 600
```
Understanding the effects of multiplying by multiples of 10 also helps students to solve problems such as $30 \times 40$, where knowing that $3 \times 4 = 12$ and $3 \times 40 = 120$ helps them to know that $30 \times 40 = 1200$.

**Developing a Variety of Computational Strategies**

Traditional approaches to teaching computation may generate beliefs about mathematics that impede further learning. These beliefs include fallacies such as the notion that only “smart” students can do math; that you must be able to do math quickly to do it well; and that math doesn’t need to be understood – you just need to follow the steps to get the answer. Research indicates that these beliefs begin to form during the elementary school years if the focus is on the mastery of standard algorithms, rather than on the development of conceptual understanding (Baroody & Ginsburg, 1986; Cobb, 1985; Hiebert, 1984).

There are numerous strategies for multiplication, which vary in efficiency and complexity. Perhaps the most complex (but not always most efficient) is the standard algorithm, which is quite difficult for students to use and understand if they have not had opportunities to explore their own strategies. For example, a common error is to misalign numbers when using the algorithm, as shown below:

```
125
\times \ 12
\underline{+ 250}
\underline{+ 125}
\underline{+ 375}
```

A student who understands multiplication conceptually will recognize that this answer is not plausible. $125 \times 10$ is 1250, so multiplying $125 \times 12$ should result in a much greater product than 375.

While the following section provides a possible continuum for teaching multiplication strategies, it is important to note that there is no “culminating” strategy – teaching the standard algorithm for multiplication should not be the ultimate teaching goal for students in the junior grades. Students need to learn the importance of looking at the numbers in the problem, and then making decisions about which strategies are appropriate and efficient in given situations.

**EARLY STRATEGIES FOR MULTIPLICATION PROBLEMS**

Students are able to solve multiplication problems long before they are taught procedures for doing so. When students are presented with problems in meaningful contexts, they rely on strategies that they already understand to work towards a solution. For example, to solve a problem that involves 8 groups of 5, students might arrange counters into groups of 5, and then skip count by 5’s to determine the total number of counters.

Students might also use strategies that involve addition.

"A baker makes 48 cookies at a time. If the baker makes 6 batches of cookies each day, how many cookies does she make?"
Two possible approaches, both using addition, are shown below:

\[
\begin{array}{c}
48 \\
+ 48 \\
96
\end{array}
\begin{array}{c}
+ 48 \\
96
\end{array}
\begin{array}{c}
144 \\
+ 48 \\
192
\end{array}
\begin{array}{c}
+ 48 \\
240
\end{array}
\begin{array}{c}
+ 48 \\
288
\end{array}
\]

As students develop concepts about multiplication, and as their knowledge of basic facts increases, they begin to use multiplicative rather than additive strategies to solve multiplication problems.

**PARTIAL PRODUCT STRATEGIES**

With partial product strategies, one or both factors in a multiplication expression are decomposed into two or more numbers, and these numbers are multiplied by the other factor. The partial products are added to determine the product of the original multiplication expression. Partial product strategies are applications of the distributive property of multiplication; for example, \(5 \times 19 = (5 \times 10) + (5 \times 9)\). The following are examples of partial product strategies.

<table>
<thead>
<tr>
<th>By Tens and Ones</th>
<th>By Place Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>To compute (38 \times 9), decompose (38) into (10 + 10 + 10 + 8), then multiply each number by (9), and then add the partial products.</td>
<td>To compute (278 \times 8), decompose (278) into (200 + 70 + 8), then multiply each number by (8), and then add the partial products.</td>
</tr>
<tr>
<td>(10 \times 9 = 90)</td>
<td>(200 \times 8 = 1600)</td>
</tr>
<tr>
<td>(10 \times 9 = 90)</td>
<td>(70 \times 8 = 560)</td>
</tr>
<tr>
<td>(10 \times 9 = 90)</td>
<td>(8 \times 8 = 64)</td>
</tr>
<tr>
<td>(8 \times 9 = 72)</td>
<td>2224</td>
</tr>
<tr>
<td>342</td>
<td></td>
</tr>
<tr>
<td>270</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An open array provides a model for demonstrating partial product strategies, and gives students a visual reference for keeping track of the numbers while performing the computations. The following example shows how \(7 \times 42\) might be represented using an open array.

\[
\begin{array}{c|c|c}
\hline
7 & 40 & 2 \\
\hline
280 &  & \\
\hline
280 + 14 = 294
\end{array}
\]
An open array can also be used to multiply a two-digit number by a two-digit number. For example, to compute $27 \times 22$, students might only decompose the 22.

Other students might decompose both 27 and 22, and use an array to show all four partial products.

Although the strategies described above rely on an understanding of the distributive property, it is not essential that students know the property by a rigid definition. What is important for them to know is that numbers in a multiplication expression can be decomposed to "friendlier" numbers, and that partial products can be added to determine the product of the expression.

**PARTIAL PRODUCT ALGORITHMS**

Students benefit from working with a partial product algorithm before they are introduced to the standard multiplication algorithm. Working with open arrays, as explained above, helps students to understand how numbers can be decomposed in multiplication. The partial product algorithm provides an organizer in which students record partial products, and then add them to determine the final product. The algorithm helps students to think about place value and the position of numbers in their proper place-value columns.
STANDARD MULTIPLICATION ALGORITHM

When introducing the standard multiplication algorithm, it is helpful for students to connect it to the partial product algorithm. Students can match the numbers in the standard algorithm to the partial products.

OTHER MULTIPLICATION STRATEGIES

The ability to perform computations efficiently depends on an understanding of various strategies, and on the ability to select appropriate strategies in different situations. When selecting a computational strategy, it is important to examine the numbers in the problem first, in order to determine ways in which the numbers can be computed easily. Students need opportunities to explore various strategies and to discuss how different strategies can be used appropriately in different situations.

It is important that students develop an understanding of the strategies through carefully planned problems. An approach to the development of these strategies is through mini-lessons involving “strings” of questions. (See Appendix 2–1: Developing Computational Strategies Through Mini-Lessons, in Volume 2: Addition and Subtraction.)

The following are some multiplication strategies for students to explore.

Compensation: A compensation strategy involves multiplying more than is needed, and then removing the “extra” at the end. This strategy is particularly useful when a factor is close to a multiple of 10. To multiply 39×8, for example, students might recognize that 39 is close to 40, multiply 40×8 to get 320, and then subtract the extra 8 (the difference between 39×8 and 40×8).

Each partial product is recorded in the algorithm:
• ones × ones
• ones × tens
• tens × ones
• tens × tens
The partial products are added to compute the final product.

27
× 22
(2 × 7)
14
(2 × 20)
40
(20 × 7)
140
(20 × 20)
200
594

27
× 22
14
40
140
400
594

27
× 22
54
540
594
The compensation strategy can be modelled using an open array.

**Regrouping:** The associative property allows the factors in a multiplication expression to be regrouped without affecting the outcome of the product. For example, \(2 \times 3 \times 6 = 2 \times (3 \times 6)\). Sometimes, when multiplying three or more factors, changing the order in which the factors are multiplied can simplify the computation. For example, the product of \(2 \times 16 \times 5\) can be found by multiplying \(2 \times 5\) first, and then multiplying \(10 \times 16\).

**Halving and Doubling:** Halving and doubling can be represented using an array model. For example, \(4 \times 4\) can be modelled using square tiles arranged in an array. Without changing the number of tiles, the tiles can be rearranged to form a \(2 \times 8\) array.

The length of the array has been doubled (4 becomes 8) and the width has been halved (4 becomes 2), but the product (16 tiles) is unchanged.

The halving-and-doubling strategy is practical for many types of multiplication problems that students in the junior grades will experience. The associative property can be used to illustrate how the strategy works.

\[
26 \times 5 = (13 \times 2) \times 5 \\
= 13 \times (2 \times 5) \\
= 13 \times 10 \\
= 1300
\]
In some cases, the halving-and-doubling process can be applied more than once to simplify a multiplication expression.

\[
12 \times 15 = 6 \times 30 = 3 \times 60 = 180
\]

When students are comfortable with halving and doubling, carefully planned activities will help them to generalize the strategy—that is, multiplying one number in the multiplication expression by a factor, and dividing the other number in the expression by the same factor, results in the same product as that for the original expression. Consequently, thirding and tripling, and fourthing and quadrupling are also possible computational strategies, as shown below.

**Thirding and Tripling**

<table>
<thead>
<tr>
<th>15 × 12</th>
<th>18 × 15</th>
<th>30 × 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 × 36 = 180</td>
<td>6 × 45 = 270</td>
<td>10 × 48 = 480</td>
</tr>
</tbody>
</table>

**Fourthing and Quadrupling**

<table>
<thead>
<tr>
<th>12 × 75</th>
<th>24 × 250</th>
<th>16 × 125</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 × 300 = 900</td>
<td>6 × 1000 = 6000</td>
<td>4 × 500 = 2000</td>
</tr>
</tbody>
</table>

**Doubling:** With the doubling strategy, a multiplication expression is simplified by reducing one of its factors by half. After computing the product for the simplified expression, the product is doubled. For example, to solve 6 × 15, the student might think “3 × 15 is 45, so double that is 90.”

An advanced form of doubling involves factoring out the twos.

For 8 × 36:
\[
2 \times 36 = 72
2 \times 72 = 144
2 \times 144 = 288
\]

Recognizing that 8 = 2 × 2 × 2 helps students know when to stop doubling.

**Developing Strategies for Multiplying Decimal Numbers**

The ability to multiply by 10 and by powers of 10 helps students to multiply decimal numbers. When students know the effect that multiplying or dividing by 10 or 100 or 1000 has on a product, they can rely on whole-number strategies to multiply decimal numbers.

To solve a problem involving 7.8 × 8, students might use the following strategy:

“Multiply 7.8 by 10, so that the multiplication involves only whole numbers. Next, multiply 78 × 8 to get 624. Then divide 624 by 10 (to “undo” the effect of multiplying 7.8 by 10 earlier). 7.8 × 8 is 62.4.”
Estimation plays an important role when multiplying two- and three-digit decimal numbers. For example, to calculate $38.8 \times 9$, students should recognize that $38.8 \times 9$ is close to $40 \times 9$, and estimate that the product will be close to 360.

Then, students perform the calculation using whole numbers, ignoring the decimal point.

\[
\begin{array}{c}
388 \\
\times 9 \\
\hline
72 \\
720 \\
2700 \\
3492
\end{array}
\]

After completing the algorithm, students refer back to their estimate to make decisions about the correct placement of the decimal point (e.g., based on the estimate, the only logical place for the decimal point is to the right of the 9, resulting in the answer 349.2).

**Developing Estimation Strategies for Multiplication**

Students should develop a range of effective estimation strategies, but they should also be aware of when one strategy is more appropriate than another. It is important for students to consider the context of the problem before selecting an estimation strategy. Students should also decide beforehand how accurate their estimate needs to be. Consider the following problem situation.

"The school secretary is placing an order for pencils for the next school year, and would like your help in figuring out how many pencils to order. She estimates that each student in the school will use about 6 pencils during the year. There will be approximately 225 students in the school. How many pencils should the school order?"

In this situation, students should estimate “on the high side” to ensure that enough, rather than too few, pencils are ordered. For example, they might multiply 250 and 6 to get an estimate of 1500.

Estimation is an important skill when solving problems involving multiplication, and there are many more strategies than simply rounding. The estimation strategies that students use for addition and subtraction may not apply to multiplication, and a firm conceptual understanding of multiplication is needed to estimate products efficiently.

The following table outlines different estimation strategies for multiplication. It is important to note that the word “rounding” is used loosely – it does not refer to any set of rules or procedures for rounding numbers (e.g., look to the number on the right; is it greater than 5 . . .).
It is important for students to know that with multiplication, rounding one factor can have a significantly different impact on the product than rounding the other. Consider the calculation $48 \times 8 = 384$. If students round 48 to 50, the estimation would be 400 (a difference of two 8’s), which is very close to the actual product. If students round 8 to 10, the estimation would be 480 (a difference of two 48’s), which is considerably farther from the actual product. Comparing the effects of rounding both factors will help develop students’ understanding of quantity and operations.

### Relating Multiplication and Division

Multiplication and division are inverse operations: multiplication involves combining groups of equal size to create a whole, whereas division involves separating the whole into equal groups. In problem-solving situations, students can be asked to determine the total number of items in the whole (multiplication), the number of items in each group (partitive division), or the number of groups (quotative division).

Students should experience problems such as the following, which allow them to see the connections between multiplication and division.

- “Samuel needs to equally distribute 168 cans of soup to 8 shelters in the city. How many cans will each shelter get?”
- “The cans come in cases of 8. How many cases will Samuel need in order to have 168 cans of soup?”

Although both problems seem to be division problems, students might solve the second one using multiplication – by recognizing that 20 cases would provide 160 cans ($20 \times 8 = 160$) and an additional case would be another 8 cans ($1 \times 8 = 8$), and therefore determining that 21 cases would provide 168 cans. With this strategy, students, in essence, decompose 168 into $(20 \times 8) + (1 \times 8)$, and then add $20 + 1 = 21$. 

### Learning About Multiplication in the Junior Grades

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Example</th>
</tr>
</thead>
</table>
| Rounding one or both numbers to the nearest multiple of 10, 100, 1000, … | $15 \times 32$ is about $15 \times 30 = 450$  
$28 \times 49$ is about $30 \times 50 = 1500$ |
| Finding friendly numbers                      | $5 \times 27$ is about $5 \times 25 = 125$                               |
| Rounding one factor up and the other factor down | $43 \times 18$ is about $40 \times 20 = 800$                             |
| Using front-end estimation  
(Note that this strategy is less accurate with multiplication than with addition.) | $125 \times 46$ is about $100 \times 40 = 4000$ (actual 5750)             |
| Finding a range (by rounding one or both factors down, then up) | $26 \times 8$ is about $20 \times 8 = 160$  
$26 \times 8$ is about $30 \times 8 = 240$  
The product is between 160 and 240. |
Providing opportunities to solve related problems helps students develop an understanding of the part-whole relationships inherent in multiplication and division situations, and enables them to use multiplication and division interchangeably depending on the problem.

**A Summary of General Instructional Strategies**

Students in the junior grades benefit from the following instructional strategies:

- experiencing a variety of multiplication problems, including equal-group and multiplicative-comparison problems;
- using concrete and pictorial models to represent mathematical situations, to represent mathematical thinking, and to use as tools for new learning;
- solving multiplication problems that serve different instructional purposes (e.g., to introduce new concepts, to learn a particular strategy, to consolidate ideas);
- providing opportunities to develop and practise mental computation and estimation strategies;
- providing opportunities to connect division to multiplication through problem solving.

The Grades 4-6 Multiplication and Division module at www.eworkshop.on.ca provides additional information on developing multiplication concepts with students. The module also contains a variety of learning activities and teaching resources.
APPENDIX 3-1: USING MATHEMATICAL MODELS TO REPRESENT MULTIPLICATION

The Importance of Mathematical Models

Models are concrete and pictorial representations of mathematical ideas, and their use is critical in order for students to make sense of mathematics. At an early age, students use models such as counters to represent objects and tally marks to keep a running count.

Standard mathematical models, such as number lines and arrays, have been developed over time and are useful as “pictures” of generalized ideas. In the junior grades, it is important for teachers to develop students’ understanding of a variety of models so that models can be used as tools for learning.

The development in understanding a mathematical model follows a three-phase continuum:

- **Using a model to represent a mathematical situation:** Students use a model to represent a mathematical problem. The model provides a “picture” of the situation.

- **Using a model to represent student thinking:** After students have discussed a mathematical idea, the teacher presents a model that represents students’ thinking.

- **Using a model as a tool for new learning:** Students have a strong understanding of the model and are able to apply it in new learning situations.

An understanding of mathematical models takes time to develop. A teacher may be able to take his or her class through only the first or second phase of a particular model over the course of a school year. In other cases, students may quickly come to understand how the model can be used to represent mathematical situations, and a teacher may be able to take a model to the third phase with his or her class.

**USING A MODEL TO REPRESENT A MATHEMATICAL SITUATION**

A well-crafted problem can lead students to use a mathematical model that the teacher would like to highlight. The following example illustrates how the use of an array as a model for multiplication might be introduced.
A teacher provides students with the following problem:

"I was helping my mother design her new rectangular patio. It will be made of square tiles. The long side of her patio will be 15 tiles long, and the short side will be 8 tiles long. How many square tiles should she buy?"

This problem was designed to encourage students to construct or draw arrays. The teacher purposefully included numbers that students could not multiply mentally.

After presenting the problem, the teacher encourages students to solve the problem in a way that makes sense to them. Some students use square tiles to recreate the patio, and then use repeated addition to determine the total number of tiles.

A student explains his strategy:

"First, I made the patio out of square tiles, and found out I had 8 rows of 15 tiles. So I added 15 eight times to get the total: 15 + 15 + 15 + 15 + 15 + 15 + 15 + 15 = 120."

Other students use similar strategies. For example, some students draw a diagram of the patio, and then add 8 fifteen times.

The teacher has not provided students with a particular model to solve the problem, but the context of the problem (creating a rectangular shape with square tiles) lends itself to using an array model. Although students used an array to represent the patio in the problem, they might not apply the array model in other multiplication problems. It is the teacher's role to help students generalize the use of the array model to other multiplication situations.

**USING A MODEL TO REPRESENT STUDENT THINKING**

Teachers can guide students in recognizing how models can represent mathematical thinking. The following example provides an illustration.

A teacher is providing an opportunity for students to develop mental multiplication strategies. He asks his students to calculate a series of multiplication questions mentally: \(6 \times 10\), \(6 \times 20\), \(6 \times 3\), \(6 \times 23\).

A student explains her strategy for solving \(6 \times 23\):

"First I multiplied 6 \times 10 to get 60. Then I multiplied 6 \times 10 again because there is a 20 in 23. I added 60 + 60 to get 120. So 6 \times 20 is 120. But it's 6 \times 23, not 6 \times 20, so I multiplied 6 \times 3 to get 18. Then I added 18 + 120 to get 138."
The teacher takes this opportunity to represent the student’s thinking by drawing an open array on the board. (The open array does not have to be drawn to scale – the dividing lines simply represent the decomposition of a factor.)

The teacher uses the open array to discuss the strategy with the class. The diagram helps students to visualize how 23 is decomposed into 10, 10, and 3; then each “part” is multiplied by 6; and then the three partial products are added together.

By representing the computational strategy using an open array, the teacher shows how the array can be used to represent mathematical thinking. Given ongoing opportunities to use open arrays to represent computational strategies and solutions to problems, students will come to “own” the model and use the open array as a tool for learning.

USING A MODEL AS A TOOL FOR NEW LEARNING

To help students generalize the use of an open array as a model for multiplication, and to help them recognize its utility as a tool for learning, teachers need to provide problems that allow students to apply and extend the strategy of partial products.

A teacher poses the following problem:

“The principal will be placing an order for school supplies, and he asked me to check the number of markers in the school’s supply cupboard. I counted 38 boxes, and I know that there are 8 markers in each box. I haven’t had time to figure out the total number of markers yet. Could you help me with this problem?”

Prior to this, the class investigated the use of open arrays and the distributive property in solving multiplication problems.

Several students use an open array to solve the problem – they decompose 38 into 30 and 8, then multiply both numbers by 8, and then add the partial products.

One student uses the array model in a different way. She explains her strategy:
“I know that 38 is close to 40, so I drew an array that was 40 long and 8 wide. I knew 8 \times 40 \text{ is } 320, but I didn’t need 40 eights – I only needed 38 eights, so I took 2 eights away at the end. 320 – 16 = 304”

The student drew the following array to solve the problem:

This student used an array to apply the distributive property but extended the use of the model to include a new compensation strategy – calculating more than is needed, and then subtracting the extra part.

In this case, the model has become a tool for learning. The student is not simply replicating a strategy used in previous problems, but instead uses it to solve a related problem in a new way.

When developing a model for multiplication, it is practical to assume that not all students will come to understand or use the model with the same degree of effectiveness. Teachers should continue to develop meaningful problems that allow students to use strategies that make sense to them. However, part of the teacher’s role is to use models to represent students’ ideas so that these models will eventually become thinking tools for students. The ability to generalize a model and use it as a learning tool takes time (possibly years) to develop.
REFERENCES


Learning Activities for Multiplication

Introduction
The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to multiplication. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or activity.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students’ understanding of mathematical concepts.
HOME CONNECTION: This section is addressed to parents or guardians, and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.
Grade 4 Learning Activity
Chairs, Chairs, and More Chairs!

OVERVIEW
In this learning activity, students solve a problem in which they determine the number of chairs arranged in a 7x24 array. The problem-solving experience provides an opportunity for students to explore a variety of multiplication strategies.

BIG IDEAS
This learning activity focuses on the following big ideas:

Operational sense: Students solve a problem involving the multiplication of a two-digit number by a one-digit number using a variety of strategies (e.g., using repeated addition, using doubling, using the distributive property). The learning activity focuses on informal strategies that make sense to students, rather than on the teaching of multiplication algorithms.

Relationships: The learning activity allows students to recognize relationships between operations (e.g., the relationship between repeated addition and multiplication). Working with arrays also helps students to develop an understanding of how factors in a multiplication expression can be decomposed to facilitate computation. For example, by applying the distributive property, 7x24 can be decomposed into (7x20) + (7x4).

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectation.
Students will:
• multiply two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., base ten materials or drawings of them, arrays), student-generated algorithms, and standard algorithms.

This specific expectation contributes to the development of the following overall expectation.
Students will:
• solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.
ABOUT THE LEARNING ACTIVITY

MATERIALS
• sheets of paper (1 per group of 3 students)
• square tiles
• Mult4.BLM1: Grid Paper (1 per group of 3 students)
• sheets of chart paper or large sheets of newsprint (1 per group of 3 students)
• markers (a few per group of 3 students)
• glue (optional)
• scissors (optional)
• sheets of paper or math journals (1 per student)
• Mult4.BLM2: Exploring Multiplication (1 per student)

MATH LANGUAGE
• repeated addition • product
• multiplication • partial products
• array • friendly number
• row • open array
• column

INSTRUCTIONAL SEQUENCING
This learning activity provides an introductory exploration of strategies for multiplying a two-digit number by a one-digit number. Before starting this activity, students should have an understanding of basic multiplication concepts (e.g., that multiplication involves combining groups of equal size), and some knowledge of basic facts (and of strategies to determine answers to unknown facts). Students should be given other opportunities to solve multiplication problems using their own strategies, before they investigate multiplication algorithms.

ABOUT THE MATH
Students are capable of solving multiplication problems before they develop an understanding of algorithms. When students apply strategies that make sense to them, they develop a deeper understanding of the operation and of different multiplication strategies.

This learning activity allows students to explore multiplication by using an array. The organization of items in rows and columns allows students to observe arrays as models of multiplication.
Dividing arrays into parts is an effective way to show how the distributive property can be applied to facilitate multiplication.

Discussions about various multiplication strategies and the use of arrays to represent multiplication are important components of this learning activity. These conversations allow students to learn strategies from one another and to recognize the power of the array as a tool for representing multiplication.

GETTING STARTED

Describe the following scenario to the class:

“The custodian at our school needs to set up chairs in the gym for a parents’ meeting. He plans to arrange the chairs in 7 rows with 24 chairs in each row. He is wondering, though, whether there will be enough chairs for 150 parents. How many chairs will there be altogether? Will there be enough chairs?”

On the board, record important information about the problem:

• 7 rows
• 24 chairs in each row
• How many chairs altogether?
• Are there enough chairs for 150 parents?

Divide the class into groups of three. Ask students to work together to solve the problem in a way that makes sense to everyone in their group. Suggest that students use materials such as square tiles and grid paper. Provide each group with a sheet of paper on which students can record their work.

WORKING ON IT

As students work on the problem, observe the various strategies they use to solve it. Pose questions to help students think about their strategies and solutions:

• “What strategy are you using to solve the problem?”
• “Why are you using this strategy?”
• “Did you change or modify your strategy? Why?”
• “How are you representing the rows of chairs? Is this an effective way to represent the chairs?”
Students might use manipulatives (e.g., square tiles), draw on grid paper, or make a diagram to represent the arrangement of chairs. Concrete arrays and pictorial arrays help students to think of and apply strategies for determining the total number of chairs.

**STRATEGIES STUDENTS MIGHT USE**

**COUNTING**
Although inefficient, counting the chairs is a strategy some students might use if they are not ready to consider the array as a representation of multiplication.

**USING REPEATED ADDITION**
The creation of a $7 \times 24$ array might prompt some students to use repeated addition – adding 7 twenty-four times, or adding 24 seven times.

**DOUBLING**
Students might use a doubling strategy similar to the following:

\[
\begin{align*}
24 & \\
24 & \\
24 & \\
24 & \\
24 & \\
24 & \\
24 & \\
\end{align*}
\]

\[48 + 48 + 24 = 168\]

\[
\begin{align*}
7 \times 10 &= 70 \\
7 \times 10 &= 70 \\
7 \times 4 &= 28 \\
70 + 70 + 28 &= 168 \\
\end{align*}
\]

**DECOMPOSING THE ARRAY (USING THE DISTRIBUTIVE PROPERTY)**
Some students might decompose $7 \times 24$ into smaller parts, then use known multiplication facts to determine the products of the smaller parts, and then add the partial products to determine the total number of chairs. Students might divide the array into two or more parts without considering whether the resulting numbers can be easily calculated.

Other students might think about ways to divide the array to work with “friendly” numbers.
USING MENTAL COMPUTATION (APPLYING THE DISTRIBUTIVE PROPERTY)

Students might think of these steps:
• 7 rows of 20 chairs is 140 chairs.
• To account for the extra 4 chairs in each row, multiply 7 × 4.
• 140 chairs + 28 chairs = 168 chairs

When students have solved the problem, provide each group with markers and a sheet of chart paper or large sheets of newsprint. Ask students to record their strategies and solutions on the paper, and to clearly demonstrate how they solved the problem. If students used grid paper, they could cut out their arrays and glue them to the sheet of paper.

Make a note of groups who might share their strategies and solutions during Reflecting and Connecting. Include groups who used various methods that range in their degree of efficiency (e.g., counting; using repeated addition; using doubling; using the distributive property without considering whether the resulting numbers can be easily calculated; using the distributive property to find friendly numbers).

REFLECTING AND CONNECTING

Reconvene the class. Ask a few groups to share their problem-solving strategies and solution, and post their work. Try to order the presentations so that students observe inefficient strategies (e.g., counting, using repeated addition) first, followed by increasingly efficient methods.

As students explain their work, ask questions that help them to describe their strategies:
• “What strategy did you use to determine the total number of chairs?”
• “Why did you use this strategy?”
• “How does your strategy work?”
• “Was your strategy easy or difficult to use? Why?”
• “Would you use this strategy if you solved a problem like this again? Why or why not?”
• “How would you change your strategy the next time?”
• “How do you know that your solution is correct?”

If students describe a mental computation strategy that is based on the distributive property, you might model their thinking by drawing an open array (i.e., an array in which the interior squares are not indicated).

Following the presentations, ask students to observe the work that has been posted, and to consider the efficiency of the various strategies. Ask:
“Which strategy, in your opinion, is an efficient strategy?”
“Why is the strategy effective in solving this kind of problem?”
“How would you explain this strategy to someone who has never used it?”

Avoid making comments that suggest that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students’ work to emphasize ideas about the distributive property:
• Arrays can be decomposed into two or more parts.
• The product of each part can be calculated and the partial products added together to determine the product of the entire array.
• An array can be decomposed into parts that provide friendly numbers, which are easy to calculate with.

Note: It is not necessary for students to define the “distributive property”, but they should learn how it can be applied to facilitate multiplication.

Provide an opportunity for students to solve a related problem. Explain that bottles of water will be set on a table for the parent meeting, and that the bottles will be arranged in 6 rows with 31 bottles in each row. Ask students to give the multiplication expression related to the problem. Record “6 \times 31” on the board.

Have students work in groups of three. (You can use the same groups as before, or form different groups.) Encourage students, in their groups, to consider the various strategies that have been discussed, and to apply a method that will allow them to solve the problem efficiently. After groups have solved the problem, ask students, independently, to record a solution on a sheet of paper or in their math journals.

ADAPTATIONS/EXTENSIONS

Encourage students to use strategies that make sense to them. Recognize that some students may need to rely on simple strategies, such as counting or using repeated addition, and may not be ready to apply more sophisticated strategies. Ensure that students use concrete materials or a drawing to represent the multiplication situation and to connect it to an array.

Guide students in using more efficient strategies when you observe that they are ready to do so. For example, students who use a counting strategy could be encouraged to use repeated addition. Provide opportunities for students who experience difficulties to work with students who can support them in understanding the arrangement in an array and how it can be divided into smaller parts.

Challenge students to solve the problem in different ways. For example, if students use an algorithm, ask them to explain how the algorithm works and the meaning of the numbers in the algorithm within the context of the problem.
ASSESSMENT

Observe students as they solve the problem, and assess how well they:
• represent and explain the problem situation (e.g., using an array made with square tiles or grid paper, using a drawing);
• apply an appropriate strategy for solving the problem;
• explain their strategy and solution;
• judge the efficiency of various strategies;
• modify or change strategies to find more efficient ways to solve the problem;
• explain ideas about the distributive property (e.g., that \(7 \times 24\) can be decomposed into \((7 \times 20) + (7 \times 4)\), and that the partial products, 140 and 28, can be added to calculate the final product).

Collect the math journals or sheets of paper on which students recorded their strategies and solutions for the water bottle problem. Observe students’ work to determine how well they apply an efficient strategy for solving the multiplication problem.

HOME CONNECTION

Send home Mult1.BLM2: Exploring Multiplication. In the letter, parents are asked to review the multiplication strategies that are modelled for them. Next they are invited to assist their child as he or she explains how to calculate the product of \(3 \times 17\).

LEARNING CONNECTION 1

How Many Fruits?

MATERIALS
• overhead transparency of Mult1.BLM3: How Many Fruits?
• overhead projector
• sheets of paper (1 per student)

Display an overhead transparency of Mult1.BLM3: How Many Fruits? Discuss how fruits are often arranged in arrays in grocery stores. Ask students to describe the different arrays of fruits in the picture.

Next, challenge students to figure out the total number of pieces of fruit in the picture. Have them record their strategies and solution on a sheet of paper.

Observe students as they solve the problem, and make note of the different methods that they use. For example, students might:
• skip count;
• use multiplication to determine the number of pieces of fruit in each tray, and then add the six partial products;
• use multiplication to determine the number of pieces of each kind of fruit, and then add the three partial products;
• multiply the number of rows by the number of pieces of fruit in each row (\(6 \times 9\)).
Ask several students to explain their strategies to the class. Include a variety of strategies, including using skip counting, using partial products, and multiplying the number of rows by the number of pieces of fruit in each row.

After a variety of strategies have been presented, have students evaluate the different methods by asking the following questions:

• “Which strategies were efficient and easy to use? Why?”
• “Which strategies are similar? How are they alike?”
• “What strategy would you use if you were to solve a problem like this again? Why?”

LEARNING CONNECTION 2
Splitting Arrays

MATERIALS
• square tiles (a large number)

Ask pairs of students to arrange tiles to form a 3 × 17 array. Instruct them to use a pencil to split the array into any two parts. For example, students might split the 3 × 17 array into a 3 × 5 array and a 3 × 12 array.

Next, ask students to explain how they could determine the total number of tiles in the 3 × 17 array using the smaller arrays. Students might suggest that they could calculate the partial product of each smaller array, and then add the partial products together. Provide time for students to apply this method using the split array that they created.

Ask several pairs of students to draw diagrams on the board, to show how they split their arrays into two parts. Include pairs who worked with partial products that were easy to calculate and then add together, such as (3 × 10) + (3 × 7) = 30 + 21 = 51, as well as those who worked with less “friendly numbers”, such as (3 × 9) + (3 × 8) = 27 + 24. Suggest that drawing each tile in the array is a time-consuming task, and encourage students to sketch open arrays instead. Explain that open arrays do not need to be drawn to scale but that the lengths and widths of the rectangles should reflect the size of numbers in the arrays.

Have each pair of students explain how they calculated the total number of tiles by adding the partial products of the smaller arrays.
After a few pairs of students have explained their work, ask:

“Which numbers were “friendly” to work with? Why were these numbers easy to calculate with?”

Repeat the activity using other arrays (e.g., $4 \times 16$ or $3 \times 23$). Observe whether or not students split arrays into parts that yield numbers that are easy to work with.

**LEARNING CONNECTION 3**

**Some More, Some Less**

**MATERIALS**

- interlocking cubes

Organize students into pairs. Instruct students to make 4 rows of 10 interlocking cubes. Ask them to tell the total number of cubes (40). On the board, record “$4 \times 10 = 40$”.

Ask: “What would you have to do to the rows of cubes you have in order to show 4 groups of 9?”

Have students remove one cube from each row, and ask them to tell the number of cubes that are in the four rows. Have students explain their strategies. Emphasize the idea that $4 \times 10$ is 40, so $4 \times 9$ is 4 less than 40 (i.e., 4 cubes were removed).

Next, have students make 4 rows of 10 cubes again, but this time, ask students to show 4 groups of 11 (by adding a cube to each row). Have students explain their strategies for calculating $4 \times 11$ (e.g., since $4 \times 10 = 40$, then $4 \times 11$ is 4 more than 40).

Have students use other rows of cubes to derive answers for other multiplication expressions. For example:

- Start with $5 \times 8$, and then calculate $5 \times 9$.
- Start with $5 \times 8$, and then calculate $5 \times 7$.
- Start with $4 \times 5$, and then calculate $4 \times 6$.

Provide practice in using mental computation. Pose pairs of expressions, such as the following, and ask students to explain how they determined the answers:

- $4 \times 10$  \hspace{1cm} $4 \times 9$
- $7 \times 10$  \hspace{1cm} $7 \times 11$
- $8 \times 5$  \hspace{1cm} $8 \times 6$
- $8 \times 5$  \hspace{1cm} $8 \times 4$

**eWORKSHOP CONNECTION**

Visit www.eworkshop.on.ca for other instructional activities that focus on multiplication concepts.

On the homepage, click “Toolkit”, in the “Numeracy” section, find “Multiplication and Division (4 to 6)”, and then click the number to the right of it.

egroup.on.ca
Exploring Multiplication

Dear Parent/Guardian:

We have been learning about the meaning of multiplication.

Here are three different ways to represent $3 \times 17$. Ask your child to explain each representation and how each representation could be used to find the answer to $3 \times 17$.

$17 + 17 + 17$

$3 \times 17$

$3 \times 10 + 3 \times 7$

Thank you for doing this activity with your child.
How Many Fruits?
Grade 5 Learning Activity
Finding the Cost of a Field Trip

OVERVIEW
In this learning activity, students solve a problem in which they determine the cost of a field trip for 29 students who each pay $20. The problem-solving experience provides an opportunity for students to explore a variety of multiplication strategies, including the use of the distributive property.

BIG IDEAS
This learning activity focuses on the following big ideas:

Operational sense: Students use a variety of strategies to solve a problem involving the multiplication of a two-digit number by a two-digit number. After solving the problem, students discuss how the distributive property can be used in multiplication.

Relationships: The activity allows students to recognize relationships between operations (e.g., the relationship between repeated addition and multiplication).

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectation.
Students will:
• multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms.

This specific expectation contributes to the development of the following overall expectation.
Students will:
• solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY
MATERIALS
• sheets of paper (1 per pair of students)
• sheets of chart paper or large sheets of newsprint (1 per pair of students)
• markers (a few per pair of students)
• sheets of paper or math journals (1 per student)
• play money (optional)
• Mult5.BLM1: 30 Packages! (1 per student)
INSTRUCTIONAL GROUPING: pairs

MATH LANGUAGE
- skip counting
- factor
- repeated addition
- product
- doubling
- partial products
- multiplication expression
- friendly number
- open array

INSTRUCTIONAL SEQUENCING
This learning activity provides an introductory exploration of strategies for multiplying a two-digit number by a two-digit number. Before starting this learning activity, students should have an understanding of multiplication concepts (e.g., that multiplication involves combining groups of equal size) and of strategies for multiplying a two-digit number by a one-digit number. Students should also be able to decompose a two-digit whole number into tens and ones (e.g., \(29 = 20 + 9\)) and should understand how to multiply numbers by multiples of 10.

ABOUT THE MATH
In this learning activity, students solve a multiplication problem using strategies that make sense to them. When students apply their own methods, they develop a deeper understanding of the operation and of the efficiency of different strategies.

This learning activity provides an opportunity for teachers to introduce the use of the distributive property in multiplication: for example, to multiply \(29 \times 20\), students might decompose 29 into \(20 + 9\), then multiply \(20 \times 20\) and \(9 \times 20\), and then add the partial products (\(400 + 180 = 580\)).

Using compensation is also an application of the distributive property. With this strategy, students multiply more than is needed, and then remove the "extra" amount. To multiply \(29 \times 20\), students might recognize that 29 is close to 30 and multiply \(30 \times 20\) to get 600. They then subtract 20 (the difference between \(30 \times 20\) and \(29 \times 20\)) to get 580.

The experience of solving problems using their own methods allows students to apply the distributive property in informal, yet meaningful, ways.

Note: It is not necessary for students to define the "distributive property", but they should learn how it can be applied to facilitate multiplication.

GETTING STARTED
Describe the following scenario to the class:

"29 students are going on a field trip to a museum. The field trip costs $20.00 per student. For this fee, each student will receive bus transportation to and from the museum, an entrance ticket to the museum, and a picnic lunch. How much will it cost for 29 students to go on the field trip?"

Divide the class into pairs. Ask students to discuss important information about the problem with their partners. Have students summarize this information. Record the following on the board:
• 29 students
• $20 per student
• cost for 29 students

WORKING ON IT

Ask students to solve the problem with their partners using a strategy that makes sense to both partners. Provide each pair of students with a sheet of paper on which they can record their work.

As students work on the problem, observe the various strategies they use to solve it. Pose questions to help students think about their strategies and solutions:

• “What strategy are you using to solve the problem?”
• “Why are you using this strategy?”
• “Did you change or modify your strategy? Why?”
• “What materials are you using? How are these materials helpful?”
• “How could you solve the problem in a different way?”
• “How could you represent your strategy so that others will know what you are thinking?”

STRATEGIES STUDENTS MIGHT USE

USING REPEATED ADDITION

Students might record $20 twenty-nine times, and repeatedly add 20 until they reach a solution.

USING SKIP COUNTING

Students might count by 20’s twenty-nine times.

USING GROUPINGS OF $100

Students might recognize that 5 × $20 = $100 and determine the cost for 25 students.

$100 (5 students)
$100 (5 students)
$100 (5 students)
$100 (5 students)
$100 (5 students)
$500 (25 students)

Students would then add the cost for 4 students (4 × $20 = $80) to determine the cost for 29 students ($500 + $80 = $580).

USING A NUMBER LINE

(continued)
USING DOUBLING
Students might continue to double the number of students and the related costs.
2 students → 2 × $20 = $40
4 students → 2 × $40 = $80
8 students → 2 × $80 = $160
16 students → 2 × $160 = $320
32 students → 2 × $320 = $640
Students will realize that 32 students is 3 more than 29 students, and might:
• subtract the cost for 3 students from $640 ($640 – $60 = $580); or
• combine the costs for 16, 8, 4, and 1 student(s), to calculate the total cost for 29 students ($320 + $160 + $80 + $20 = $580).

USING A T-CHART

<table>
<thead>
<tr>
<th>Number of Students</th>
<th>Field Trip Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$40</td>
</tr>
<tr>
<td>4</td>
<td>$80</td>
</tr>
<tr>
<td>8</td>
<td>$160</td>
</tr>
<tr>
<td>16</td>
<td>$320</td>
</tr>
<tr>
<td>32</td>
<td>$640</td>
</tr>
<tr>
<td>29</td>
<td>$580</td>
</tr>
</tbody>
</table>

APPLYING THE DISTRIBUTIVE PROPERTY
Some students might apply the distributive property in any of the following ways:
• Decompose 29 into smaller parts, then multiply each part by $20, and then add the partial products.

**Example 1:**
29 × $20 is the same as (10 + 10 + 9) × $20.
(10 × $20) + (10 × $20) + (9 × $20) = $200 + $200 + $180 = $580

**Example 2:**
29 × $20 is the same as (20 + 9) × $20.
(20 × $20) + (9 × $20) = $400 + $180 = $580

• Use an open array to model partial products.
• Use a partial product algorithm.

\[
\begin{array}{c}
29 \\
\times 20 \\
\hline
180 \\
(20 \times 9) \\
400 \\
(20 \times 20) \\
\hline
580
\end{array}
\]

USING COMPENSATION
Some students might recognize that 29 is close to 30. They might determine the total cost of the field trip for 30 students, and then subtract 20 (the extra $20 fee for one student), to calculate the total cost of the field trip for 29 students.

\[
30 \times 20 = 600 \\
600 - 20 = 580
\]

This compensation strategy can be modelled using an open array.

USING A STANDARD ALGORITHM
Students might have learned the procedures in using a standard algorithm. Ask students to explain the meaning of each step in the algorithm. If they are unable to do so, suggest that they use a strategy that they can explain.

After students have solved the problem, provide each pair with markers and a sheet of chart paper or a large sheet of newsprint. Ask students to record their strategies and solutions on the paper, and to clearly demonstrate how they solved the problem.

Make a note of groups that might share their strategies and solutions during Reflecting and Connecting. Include groups who used various methods that range in their degree of sophistication (e.g., using repeated addition, using doubling, applying the distributive property).
REFLECTING AND CONNECTING

Reconvene the class. Ask a few groups to share their problem-solving strategies and solution, and post their work. Try to order the presentations so that students observe inefficient strategies (e.g., using repeated addition, using skip counting) first, followed by more efficient methods.

As students explain their work, ask questions that probe their thinking:

- "How did you determine the total cost of the field trip for 29 students?"
- "Why did you use this strategy? How did the numbers in the problem help you choose a strategy?"
- "Was your strategy easy or difficult to use? Why?"
- "Would you use this strategy if you solved another problem like this again? Why or why not?"
- "How would you change your strategy the next time?"
- "How did you record your strategy?"
- "Is your strategy similar to another strategy? Why or why not?"
- "How do you know that your solution is correct?"

Following the presentations, ask students to observe the work that has been posted, and to consider the efficiency of the various strategies. Ask:

- "Which strategy, in your opinion, is an efficient strategy?"
- "Why is the strategy effective in solving this kind of problem?"
- "How would you explain this strategy to someone who has never used it?"

Avoid making comments that suggest that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students’ work to emphasize important ideas about the distributive property:

- Two-digit factors can be decomposed into parts to facilitate multiplication (e.g., 29 can be decomposed into 20 and 9, which are friendly numbers that are easy to multiply). Each part is multiplied by the other factor in the multiplication expression (20 × 20 = 400, 9 × 20 = 180), and then the partial products are added to calculate the total product (400 + 180 = 580).
- An open array shows how factors in a multiplication expression can be decomposed into two or more parts. Partial products are recorded on the open array, and then added to calculate the total product.
- The parts in an open array can be represented in a partial product algorithm.
Provide an opportunity for students to solve a related problem. Explain that along with the 29 students, 7 adults will be going on the field trip to help supervise. The fee for each adult is also $20 per person. Ask students to give the multiplication expression related to the problem. Record “36 × 20” on the board.

Have students work in pairs. Encourage them to consider the various strategies that have been discussed and to apply a method that will allow them to solve the problem efficiently. After pairs have solved the problem, ask students to independently record a solution on a sheet of paper or in their math journals.

ADAPTATIONS/EXTENSIONS

Encourage students to use strategies that make sense to them. Recognize that some students may need to rely on simple strategies, such as repeated addition and skip counting, and may not be ready to apply more sophisticated strategies.

Some students may benefit from using play money (e.g., $20 bills) to represent the problem and to find a strategy. Scaffold the problem by asking:

- “How many $20 bills will be needed for 1 student?”
- “How many $20 bills will be needed for 5 students? What will the cost be for 5 students? For 10 students? For 20 students? For 29 students?”

Guide students in using more efficient strategies when you observe that they are ready to do so. Provide opportunities for these students to work with classmates who can demonstrate the use of more efficient strategies.

Challenge students to solve the problem in different ways. For example, if students use an algorithm, ask them to explain how the algorithm works and the meaning of the numbers in the algorithm within the context of the problem.

ASSESSMENT

Observe students as they solve the problem, and assess how well they:

- represent and explain the problem;
- apply an appropriate strategy for solving the problem;
- explain their strategy and solution;
- judge the efficiency of various strategies;
- modify or change strategies to find more efficient ways to solve the problem;
- explain ideas about the distributive property (e.g., that 29 × 20 can be decomposed into (20 × 20) + (9 × 20), and that the partial products, 400 and 180, can be added to determine the final product).

Collect the math journals or sheets of paper on which students recorded their strategies for determining the cost of a trip for 29 students and 7 adults. Observe students’ work to determine how well they apply an efficient strategy for solving the multiplication problem.
HOME CONNECTION
Send home Mult5.BLM1: 30 Packages! In this Home Connection activity, students are asked to find packages that contain between 10 and 50 items, and then calculate the number of items that would be in 30 packages.

LEARNING CONNECTION 1
Applying the Distributive Property

MATERIALS
• sheets of paper (1 per student)

Provide opportunities for students to use the distributive property to simplify multiplication. Record “27 × 30” on the board, and challenge students to calculate the product mentally. (Allow students to keep track of some of their calculations on paper, but encourage them to do most of the calculations in their head.) Have students explain their strategies.

Discuss the steps in the following strategy:
• 27 can be decomposed into 20 and 7.
• Each part can be multiplied by 30 (20 × 30 = 600, 7 × 30 = 210).
• The partial products can be added to calculate the total product (600 + 210 = 810).

Illustrate the strategy using an open array.

Provide other multiplication expressions for students to calculate mentally:
• 43 × 30
• 58 × 40
• 62 × 50

Have students draw open arrays on the board to represent the calculation of each expression using the distributive property.
LEARNING CONNECTION 2
What Would the Array Look Like?

MATERIALS
- sheets of paper (1 per student)

Discuss with students how it is impractical to use square tiles to create arrays that involve larger numbers (e.g., to show a $4 \times 30$ array). Have students suggest other ways to show an array with large numbers (e.g., using grid paper, using an open array). Talk about the advantages of using open arrays (e.g., they are easy to draw; they can represent large numbers; they are uncluttered).

On the board, record “$4 \times 30$”. Ask students to imagine what the open array would look like, and then have them draw an open array on a sheet of paper. Draw an example on the board, and explain that an open array does not need to be drawn to scale but that the length and width of the rectangle should reflect the size of the numbers in the array.

Ask students to give the product of $4 \times 30$, and record “120” on the array.

Next, record “$4 \times 31$” on the board, and ask students to explain how the open array would compare with the one for $4 \times 30$. Instruct students to add a section to their $4 \times 30$ array to show $4 \times 31$.

Ask students to explain how the open array can help them calculate $4 \times 31$ (e.g., $3 \times 40 = 120$ and $4 \times 1 = 4$, so $4 \times 31$ is $120 + 4$, or 124).

Repeat the activity by having students draw open arrays for the following pairs of multiplication expressions:
- $3 \times 20$ 3 $\times$ 22
- $5 \times 40$ 5 $\times$ 43
- $7 \times 30$ 7 $\times$ 35

For each pair, ask students to explain how the open array for the first multiplication expression can help them calculate the product for the second expression.
LEARNING CONNECTION 3
Exploring the Commutative Property of Multiplication

MATERIALS
- square tiles (12 per pair of students)
- Mult5.BLM2: Large Grid Paper (a few sheets per pair of students)
- scissors (1 pair per pair of students)
- sheets of paper (1 per student)

Provide an opportunity for students to represent and discuss the commutative property of multiplication. Give 12 square tiles to each pair of students, and ask students to create as many different arrays as possible. Instruct them to make paper cut-outs of each array using Mult5.BLM2: Large Grid Paper.

After students have finished cutting out the different arrays, ask them to discuss observations about the arrays with their partners. Provide a few minutes for discussion, and then reconvene the whole class. Invite students to share their observations with the large group.

On the board, post a $2 \times 6$ array and a $6 \times 2$ array, and record the multiplication expression next to each.

Have students compare the two arrays and discuss:
- Both arrays involve the same factors (2 and 6), but their order is different in each array.
- The $2 \times 6$ and the $6 \times 2$ arrays are congruent (i.e., they both involve the same arrangement of squares).
- One array is a rotation of the other.

Next, post a $3 \times 4$ array and a $4 \times 3$ array, and record the multiplication expression next to each. Again, have students share their observations about the two arrays.

Hold a similar discussion about the $1 \times 12$ and the $12 \times 1$ arrays.

Ask:
- “What do the pairs of arrays show about multiplication?” (The factors in a multiplication expression can be placed in any order.)
- “How could you determine the answer to $9 \times 3$ if you know the answer to $3 \times 9$?”
Provide each student with a sheet of paper. Challenge them to prove on paper that \(4 \times 8\) has the same product as \(8 \times 4\). Observe students’ work, to assess how well they understand the commutative property of multiplication.

Note: It is not necessary for students to define the “commutative property”, but they should learn how it can be applied to facilitate multiplication.

**eWORKSHOP CONNECTION**

Visit www.eworkshop.on.ca for other instructional activities that focus on multiplication concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Multiplication and Division (4 to 6)”, and then click the number to the right of it.
30 Packages!

Dear Parent/Guardian:

We have been learning about multiplication.

Have your child work on the following activity. Ask your child to explain his or her work.

Find packages in your home that contain between 10 and 50 items (for example, bottles of water in a case, eggs in a carton, tea bags in a box). Find the total number of items in the package.

Next, imagine that you have 30 packages. Determine the total number of items in 30 packages. Show your work in the space below.

Explain your work to a family member.

Thank you for doing this activity with your child.
Grade 6 Learning Activity
Shopping for Puppy Food

OVERVIEW
In this learning activity, students use a variety of multiplicative strategies (e.g., using repeated addition, using doubling, using proportional reasoning) to calculate and compare the costs of 24 cans of puppy food at three different stores.

BIG IDEAS
This learning activity focuses on the following big ideas:

Operational sense: Students use a variety of strategies to solve a problem involving multiplicative reasoning and discuss the efficiency of various strategies.

Relationships: Students compare costs expressed as decimal numbers.

Proportional reasoning: The learning activity provides an opportunity for students to apply proportional reasoning to determine the cost of 24 cans of puppy food.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations.

Students will:
• represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);
• multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators (e.g., calculate 4 × 1.4 using base ten materials; calculate 5.6 ÷ 4 using base ten materials);
• represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation.

These specific expectations contribute to the development of the following overall expectations.

Students will:
• read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;
• solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;
• demonstrate an understanding of relationships involving percent, ratio, and unit rate.
ABOUT THE LEARNING ACTIVITY

MATERIALS
• sheets of paper (1 per group of 2 or 3 students)
• sheets of chart paper or large sheets of newsprint (1 per group of 2 or 3 students)
• markers (a few per group of 2 or 3 students)
• sheets of paper or math journals (1 per student)
• Mult6.BLM1: 2 for ..., 3 for ... (1 per student)

MATH LANGUAGE
• repeated addition
• doubling
• factor
• product
• partial products
• rate

INSTRUCTIONAL SEQUENCING
Prior to this learning activity, students should have had opportunities to add decimal numbers to hundredths (e.g., money amounts), and to represent simple multiplicative relationships involving rates (e.g., “If a box contains 6 markers, then 6 boxes contain 36 markers.”).

ABOUT THE MATH
An understanding of multiplicative situations is critical in the development of students’ mathematical thinking. This understanding allows students to:
• express relationships between quantities (e.g., “There is 3 times as much snow on the ground today as yesterday.”);
• solve problems involving rates (e.g., “If 2 books cost $7.25, then 4 books cost $14.50.”);
• determine equivalent fractions (e.g., “If young children sleep about 1/3 of a day, then they sleep about 8 hours or 8/24 of a day.”);
• reason proportionally (e.g., “A DJ plays 3 fast songs for every slow song, so if the DJ plays 4 slow songs, then she plays 12 fast songs.”).

This learning activity provides an opportunity for students to solve a problem involving multiplicative relationships. The experience of using various informal strategies (e.g., using repeated addition, using doubling, using ratio tables) allows students to comprehend multiplicative situations, and prepares them for learning more formal strategies in subsequent grades.

GETTING STARTED
Explain the following situation to the class:

“My friend called me last evening because he was very excited about getting a new puppy. My friend explained that the puppy needs special food to help it to grow up to be a healthy dog. He told me that there are three stores in his neighbourhood that sell the special puppy food.”
On the board, record the store names and prices for the puppy food:
- Pat’s Pet Emporium: $0.80 per can
- Pet-o-rama: $9.40 for a dozen cans
- Petmania: $2.55 for three cans

Pose the problem: “My friend wants to buy 24 cans of puppy food. How much will he pay for the puppy food at each store? At which store will he get the best price?”

Ensure that students understand the problem. Ask:
- “What do you need to find out?”
- “What information will you need to use to solve the problem?”

WORKING ON IT
Organize students into groups of two or three. Encourage them to work collaboratively to solve the problem. Provide each group with a sheet of paper on which students can record their work.

Observe students as they solve the problem. Ask questions that help students think about their problem-solving strategies and solutions:
- “How are you solving the problem?”
- “What part of this problem is easy for you to solve? What is difficult?”
- “How can you determine the cost of 24 cans at Pat’s Pet Emporium? Pet-o-rama? Petmania?”
- “What other strategies can you use to determine the cost of 24 cans at each store?”
- “How can you record your solution so that others will understand how you solved the problem?”

STRATEGIES STUDENTS MIGHT USE

USING DOUBLING
Many students will double $9.40 (the cost of a dozen cans) to calculate the cost of 24 cans at Pet-o-rama ($18.80).

To determine the cost of 24 cans at Pat’s Pet Emporium and at Petmania, students might use a variety of strategies.

(In the following examples, decimal points have been included in the calculations. It is also acceptable for students to perform the calculations using whole numbers, and then add dollar signs and decimal points to the results to indicate monetary amounts.)

USING REPEATED ADDITION
Students might repeatedly add the cost of single cans until they determine the cost of 24 cans (e.g., adding 0.80 twenty-four times to calculate the cost of 24 cans at Pat’s Pet Emporium).
USING DOUBLING
Students might repeatedly double the number of cans and their costs. For example, for Pat’s Pet Emporium:

\[
\begin{align*}
0.80 + 0.80 &= 1.60 \text{ (2 cans)} \\
1.60 + 1.60 &= 3.20 \text{ (4 cans)} \\
3.20 + 3.20 &= 6.40 \text{ (8 cans)} \\
6.40 + 6.40 &= 12.80 \text{ (16 cans)} \\
\end{align*}
\]

$12.80 \text{ (the cost of 16 cans)} + $6.40 \text{ (the cost of 8 cans)} = $19.20 \text{ (the cost of 24 cans)}

USING A RATIO TABLE
Students might use a ratio table to generate the cost of 24 cans. For example, to determine the cost of 24 cans at Petmania, students might double the number of cans and the costs of the cans until they determine the cost of 24 cans.

<table>
<thead>
<tr>
<th>Number of cans</th>
<th>3</th>
<th>6</th>
<th>12</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$2.55</td>
<td>$5.10</td>
<td>$10.20</td>
<td>$20.40</td>
</tr>
</tbody>
</table>

USING PARTIAL PRODUCTS
To calculate the cost of 24 cans at Pat’s Pet Emporium, students might decompose 24 into 10, 10, and 4, then multiply each number by $0.80, and then add the partial products.

Cost of 10 cans \(\rightarrow\) 10 \(\times\) $0.80 = $8.00
Cost of 4 cans \(\rightarrow\) 4 \(\times\) $0.80 = $3.20
Cost of 24 cans \(\rightarrow\) $8.00 + $8.00 + $3.20 = $19.20

(continued)
USING A MULTIPLICATION ALGORITHM

Students might use an algorithm to calculate the cost.

\[
\begin{array}{c}
24 \\
\times 0.8 \\
\hline
19.2
\end{array}
\]

Cost of 24 cans = $19.20

USING PROPORTIONAL REASONING

To determine the cost of 24 cans at Petmania, students might recognize that 3 is a factor of 24 (3 \times 8 = 24) and use this multiplicative relationship to reason proportionally; that is, multiply $2.55 (the cost of 3 cans) by 8 to calculate the cost of 24 cans.

When students have solved the problem, provide each group with markers and a sheet of chart paper or large sheets of newsprint. Ask students to record their strategies and solutions on the paper, and to clearly demonstrate how they solved the problem.

Make a note of the various strategies used by students, and consider which groups might present their strategies during Reflecting and Connecting. Aim to include a variety of strategies that range in their degree of efficiency (e.g., using repeated addition, using doubling, using partial products, using proportional reasoning).

REFLECTING AND CONNECTING

Reconvene the class after the students have solved the problem. Begin a discussion by asking general questions about the problem-solving experience:

• “How did your group decide how to solve this problem?”
• “What was easy about solving this problem?”
• “What was difficult about solving the problem?”

Have a few groups present their strategies for determining the cost of 24 cans at the three pet stores, and for comparing the prices.

As students explain their work, ask questions that probe their thinking, and encourage them to explain their strategies:

• “How did you determine the cost of 24 cans at each store?”
• “Why did you use this strategy?”
• “What worked well with this strategy? What did not work well?”
• “Would you use this strategy if you solved another problem like this again? Why or why not?”
• “How would you change your strategy the next time?”
• “How did you record your strategy?”
• “Which store offers the best price? How do you know?”
Following the presentations, encourage students to consider the efficiency of the various strategies that have been presented. Ask:

• "In your opinion, which strategy worked well?"
• "Why is the strategy effective in solving this kind or problem?"
• "How would you explain this strategy to someone who has never used it?"

Avoid making comments that suggest that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Pose the following problem:

“A flyer for a pet store advertises a special – 4 cans of puppy food for $2.90. What is the cost of 24 cans?”

Have students work independently to solve the problem. Encourage them to think back to the different strategies presented by classmates, and to use an efficient strategy that makes sense to them. Have students show their strategies and solution on a sheet of paper or in their math journals.

ADAPTATIONS/EXTENSIONS

Some students may benefit from solving a version of the problem that involves simpler numbers (e.g., determining the best buy given $1.00 per can, $9.50 for 10 cans, $2.20 for 2 cans).

For students who require a greater challenge, extend the problem by having them determine the amount of money saved if a person buys 72 cans at Pet-o-rama rather than at Petmania.

ASSESSMENT

Observe students as they solve the problem:

• How efficient are students’ strategies for determining the cost of 24 cans at each pet store?
• How well do students apply proportional reasoning?
• How accurate are students’ calculations?
• Are students able to compare prices?
• How well do students explain their strategies and solutions?
• Are students able to judge the efficiency of various strategies?

Examine students’ solutions for the problem posed at the conclusion of Reflecting and Connecting. Assess how well students selected and applied efficient strategies to solve that problem.

HOME CONNECTION

Send home Multi BLM: 2 for ..., 3 for .... In this Home Connection activity, parents and students discuss the prices of grocery store items that are sold at rates, such as “3 for $1.49”.

Grade 6 Learning Activity: Shopping for Puppy Food
LEARNING CONNECTION 1
How Much Larger Is That Letter?

MATERIALS
• Mult6.BLM2: How Much Larger Is That Letter? (1 per student)

Provide each student with a copy of Mult6.BLM2: How Much Larger Is That Letter? and have them complete the three activities described. These activities help students to connect ideas about scaling and proportions.

LEARNING CONNECTION 2
Ratios and Rates are Everywhere!

MATERIALS
• newspapers, magazines, store flyers (brought to class by students)
• scissors (1 pair per student)
• large sheets of paper (1 per group of 3 or 4 students)
• glue

Discuss the meaning of ratio and rate. For example, a ratio is a comparison of similar types of things, as in “3 cars to 4 trucks” (both cars and trucks are vehicles); whereas a rate involves a comparison of two items with different units, as in “60 kilometres per hour” or “6 cans for $2.99”. Have students give examples of ratios and rates.

Arrange students in groups of three or four. Instruct them to create a collage by cutting out examples of ratios and rates in newspapers, magazines, and store flyers, and gluing them onto a large sheet of paper. Encourage students to organize their examples in some way, such as according to “ratios” and “rates”, by kinds of items, or in the ways used to express ratios and rates (e.g., 3 for $0.99, 3/$0.99). Have groups present their examples. Discuss how rates and ratios are used, and the various ways to express them.

LEARNING CONNECTION 3
Estimating the Cost of Breakfast

MATERIALS
• Mult6.BLM3: Estimating the Cost of Breakfast (1 per student)


Discuss the problem, and explain that students are to estimate Lenore’s cost for all the breakfast foods. Encourage students to use estimation strategies that make sense to them.

Have pairs of students present their findings to the class. Discuss the different estimation strategies used by students.
LEARNING CONNECTION 4
Using the Associative Property to Simplify Multiplication

On the board, record “4 \times 7 \times 5”. Ask students to mentally calculate the answer by multiplying 4 \times 7 first, and then multiplying 28 \times 5.

Next, record “4 \times 5 \times 7” on the board, and again, have students multiply the factors from left to right (4 \times 5 = 20, 20 \times 7 = 140).

Ask students to compare the multiplication expressions and the answers. Emphasize the idea that the factors in both expressions are the same but presented in a different order, and that the product is the same for both expressions.

Ask students to explain which expression was easier to calculate. Students might comment that the multiplications in the second expression (4 \times 5 and 20 \times 7) were easier to perform than the multiplications in the first expression (4 \times 7 and 28 \times 5).

Have students calculate other pairs of expressions:

- 2 \times 8 \times 5
- 4 \times 9 \times 5
- 5 \times 7 \times 8

Next, have students propose a strategy for multiplying three or more factors (e.g., the order of the factors can be changed to facilitate multiplication because changing the order of the factors does not change the product).

Have students apply the strategy to calculate other multiplication expressions, such as the following:

- 8 \times 6 \times 5
- 5 \times 9 \times 6

LEARNING CONNECTION 5
Halving and Doubling

MATERIALS

- square tiles (24 for each pair of students)
- sheets of paper (1 per student)

Record the following table on the board or on chart paper:
Arrange students in pairs. Provide each pair with 24 square tiles, a sheet of paper, and a pencil. Have pairs copy the table onto their paper and then use the tiles to create the arrays listed. Instruct students to record the width and length of each array on their table.

Talk about the activity after students have completed their tables. Discuss how the number of tiles remains the same for each array. Have students explain how they created each new array from the previous one. For example, students may have slid the top half or bottom half of an array to create a new array.

Ask students to describe patterns in the table. Emphasize the idea that the width of each array is half the width of the previous array, while the length is double the length of the previous array.

Repeat the activity by having students create arrays for $8 \times 2$, $4 \times 4$, $2 \times 8$, and $1 \times 16$, and have them record their findings in a table. Discuss how the width is halved and the length is doubled with each new array.

Ask students how they might use a halving-and-doubling strategy to calculate $4 \times 17$. Students might suggest that they could halve 4 and double 17 to create $2 \times 34$. Discuss how $2 \times 34$ is easier to calculate mentally than $4 \times 17$.

Have students practise halving and doubling with other multiplication expressions, such as $16 \times 5$, $24 \times 5$, $12 \times 25$, $18 \times 25$, $18 \times 50$, and $42 \times 50$. Discuss situations in which the strategy is useful for performing mental calculations.

Extend the activity by having students investigate whether doubling first and then halving is a workable strategy. Have them propose multiplication expressions for which a doubling-and-halving strategy would be useful.
eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on multiplication concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Multiplication and Division (4 to 6)”, and then click the number to the right of it.

eworkshop.on.ca
Dear Parent/Guardian,

We are learning about multiplication.

Grocery store items are often sold in quantities of 2 or 3 (for example, 2 cans for $1.99, 3 bottles for $3.49).

Look through a grocery store flyer with your child, and find examples of food that are sold as “2 for …”, “3 for …”, “6 for …”, and so on.

Select an item and have your child use a calculator to find the cost of multiple items. For example, if the price of tomato soup is 3 cans for $1.39, you might use a table to record the price of 3, 6, 9, 12, and 15 cans.

<table>
<thead>
<tr>
<th>Cans</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$1.39</td>
<td>$2.78</td>
<td>$4.17</td>
<td>$5.56</td>
<td>$6.95</td>
</tr>
</tbody>
</table>

Next, select a different item with your child. Without your child watching, use a calculator to determine the cost of several of the items. Tell your child the cost of several items, and have him or her estimate the number of items. For example, if containers of yogurt are 2 for $4.89, you might calculate the cost of 12 containers, and ask: “How many containers could I buy for $29.34?” Have your child use a calculator to check his or her estimate.

Thank you for doing this activity with your child.
How Much Larger Is That Letter?

• Find a way to determine how much larger the second letter in each row is than the first letter. Explain your method to a partner.

H H
E E
T T

• Choose a letter and print it on a piece of paper. Next, print a second letter that is proportionally larger. Remember to increase both the height and the width of the letter by the same factor. For example, if you quadruple the height, you must also quadruple the width.

• Ask a partner to figure out how much larger your second letter is than your first letter. Have your partner explain his or her thinking.
Estimating the Cost of Breakfast

Lenore is a caterer. To help her plan a breakfast for 120 people, she made a table that shows the food she will serve, the amount of food required per person, and the price she needs to pay for each kind of food.

<table>
<thead>
<tr>
<th>Food</th>
<th>Amount per Person</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>orange juice</td>
<td>1 juice box</td>
<td>pack of 3 boxes/$1.29</td>
</tr>
<tr>
<td>eggs</td>
<td>2 eggs</td>
<td>$1.49 per dozen</td>
</tr>
<tr>
<td>croissants</td>
<td>1 croissant</td>
<td>6/$2.49</td>
</tr>
<tr>
<td>yogurt</td>
<td>1 container</td>
<td>pack of 8 containers/$4.89</td>
</tr>
</tbody>
</table>

Estimate how much Lenore needs to pay for all the food.
Number Sense and Numeration, Grades 4 to 6

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

Volume 4
Division

Ontario Education excellence for all
Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.
Number Sense and Numeration, Grades 4 to 6

Volume 4
Division

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6
INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of The Ontario Curriculum, Grades 1–8: Mathematics, 2005. This guide provides teachers with practical applications of the principles and theories behind good instruction that are elaborated on in A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006.

The guide comprises the following volumes:
• Volume 1: The Big Ideas
• Volume 2: Addition and Subtraction
• Volume 3: Multiplication
• Volume 4: Division
• Volume 5: Fractions
• Volume 6: Decimal Numbers

The present volume – Volume 4: Division – provides:
• a discussion of mathematical models and instructional strategies that support student understanding of division;
• sample learning activities dealing with division for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume also contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by “eLearning modules” that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pp. 44, 55, and 68).
Relating Mathematics Topics to the Big Ideas

The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the “big ideas” of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005, p. 4)

In planning mathematics instruction, teachers generally develop learning opportunities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- operational sense
- proportional reasoning
- relationships

Each of the big ideas is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students.
The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

**Problem Solving:** Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

**Reasoning and Proving:** The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

**Reflecting:** Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

**Selecting Tools and Computational Strategies:** Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.

**Connecting:** The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect
procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

**Representing:** The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students’ own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

**Communicating:** Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

**Addressing the Needs of Junior Learners**

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The following table outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.
### Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
</table>
| **Intellectual development** | Generally, students in the junior grades:  
• prefer active learning experiences that allow them to interact with their peers;  
• are curious about the world around them;  
• are at a concrete operational stage of development, and are often not ready to think abstractly;  
• enjoy and understand the subtleties of humour. | The mathematics program should provide:  
• learning experiences that allow students to actively explore and construct mathematical ideas;  
• learning situations that involve the use of concrete materials;  
• opportunities for students to see that mathematics is practical and important in their daily lives;  
• enjoyable activities that stimulate curiosity and interest;  
• tasks that challenge students to reason and think deeply about mathematical ideas. |
| **Physical development** | Generally, students in the junior grades:  
• experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys);  
• are concerned about body image;  
• are active and energetic;  
• display wide variations in physical development and maturity. | The mathematics program should provide:  
• opportunities for physical movement and hands-on learning;  
• a classroom that is safe and physically appealing. |
| **Psychological development** | Generally, students in the junior grades:  
• are less reliant on praise but still respond well to positive feedback;  
• accept greater responsibility for their actions and work;  
• are influenced by their peer groups. | The mathematics program should provide:  
• ongoing feedback on students’ learning and progress;  
• an environment in which students can take risks without fear of ridicule;  
• opportunities for students to accept responsibility for their work;  
• a classroom climate that supports diversity and encourages all members to work cooperatively. |
| **Social development** | Generally, students in the junior grades:  
• are less egocentric, yet require individual attention;  
• can be volatile and changeable in regard to friendship, yet want to be part of a social group;  
• can be talkative;  
• are more tentative and unsure of themselves;  
• mature socially at different rates. | The mathematics program should provide:  
• opportunities to work with others in a variety of groupings (pairs, small groups, large group);  
• opportunities to discuss mathematical ideas;  
• clear expectations of what is acceptable social behaviour;  
• learning activities that involve all students regardless of ability. |

(continued)
### Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moral and ethical development</td>
<td>Generally, students in the junior grades: • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask “why” questions; • begin to consider others’ points of view.</td>
<td>The mathematics program should provide: • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas and evaluate the ideas of others.</td>
</tr>
</tbody>
</table>

(Adapted, with permission, from Making Math Happen in the Junior Grades. Elementary Teachers’ Federation of Ontario, 2004.)
LEARNING ABOUT DIVISION IN THE JUNIOR GRADES

Introduction
Students’ understanding of division concepts and strategies is developed through meaningful and purposeful problem-solving activities. Solving a variety of division problems and discussing various strategies and methods helps students to recognize the processes involved in division, and allows them to make connections between division and addition, subtraction, and multiplication.

PRIOR LEARNING
Initial experiences with division in the primary grades often involve sharing objects equally. For example, students might be asked to show how 4 children could share 12 boxes of raisins fairly. Using 12 counters to represent the boxes, students might divide the counters into 4 groups while counting out, “One, two, three, four, one, two, three, four, …” until all the “boxes” have been distributed.

Students in the primary grades also apply their understanding of addition, subtraction, and multiplication to solve division problems. Consider the following problem.

“Chad has 28 dog treats. If he gives Rover 4 dog treats each day, for how many days will Rover get treats?”

Using addition: Students might repeatedly add 4 until they get to 28, and then count the number of times they added 4. Students often use drawings to help them keep track of the number of repeated additions they make.
Using subtraction: Students might start with 28 counters and remove them in groups of 4. Later, students make connections to repeated subtraction (e.g., repeatedly subtracting 4 from 28 until they get to 0, and then counting the number of times 4 was subtracted).

Using multiplication: Students might use their knowledge of multiplication. For example, “Rover gets 4 treats each day. Since $4 \times 7 = 28$, Rover will get treats for 7 days.”

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

In the junior grades, instruction should focus on developing students’ understanding of division concepts and meaningful computational strategies, rather than on having students memorize the steps in algorithms.

Development of division concepts and computational strategies should be rooted in meaningful experiences that allow students to model multiplicative relationships (i.e., represent a quantity as a combination of equal groups), and encourage them to develop and apply a variety of strategies.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to division, listed in the following table.

<table>
<thead>
<tr>
<th>Curriculum Expectations Related to Division, Grades 4, 5, and 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>By the end of Grade 4, students will:</strong></td>
</tr>
<tr>
<td><strong>Overall Expectation</strong></td>
</tr>
<tr>
<td>• solve problems involving the addition, subtraction, multiply-</td>
</tr>
<tr>
<td>cation, and division of single- and multidigit whole numbers,</td>
</tr>
<tr>
<td>and involving the addition and subtraction of decimal numbers</td>
</tr>
<tr>
<td>to tenths and money amounts, using a variety of strategies.</td>
</tr>
<tr>
<td><strong>Specific Expectations</strong></td>
</tr>
<tr>
<td>• multiply to $9 \times 9$ and divide to $81 - 9$, using a variety of mental strategies;</td>
</tr>
<tr>
<td>• multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100 using mental strategies;</td>
</tr>
<tr>
<td>• divide two-digit whole numbers by one-digit whole numbers, using a variety of tools and student-generated algorithms.</td>
</tr>
<tr>
<td><strong>By the end of Grade 5, students will:</strong></td>
</tr>
<tr>
<td><strong>Overall Expectation</strong></td>
</tr>
<tr>
<td>• solve problems involving the multiplication and division of</td>
</tr>
<tr>
<td>multidigit whole numbers, and involving the addition and sub-</td>
</tr>
<tr>
<td>traction of decimal numbers to hundredths, using a variety of</td>
</tr>
<tr>
<td>strategies.</td>
</tr>
<tr>
<td><strong>Specific Expectations</strong></td>
</tr>
<tr>
<td>• divide three-digit whole numbers by one-digit whole numbers,</td>
</tr>
<tr>
<td>using concrete materials, estimation, student-generated algo-</td>
</tr>
<tr>
<td>rithms, and standard algorithms;</td>
</tr>
<tr>
<td>• multiply decimal numbers by 10, 100, 1000, and 10 000, and</td>
</tr>
<tr>
<td>divide decimal numbers by 10 and 100, using mental strategies;</td>
</tr>
<tr>
<td>• use estimation when solving problems involving the addition,</td>
</tr>
<tr>
<td>subtraction, multiplication, and division of whole numbers, to</td>
</tr>
<tr>
<td>help judge the reasonableness of a solution.</td>
</tr>
<tr>
<td><strong>By the end of Grade 6, students will:</strong></td>
</tr>
<tr>
<td><strong>Overall Expectation</strong></td>
</tr>
<tr>
<td>• solve problems involving the multiplication and division of</td>
</tr>
<tr>
<td>whole numbers, and the addition and subtraction of decimal</td>
</tr>
<tr>
<td>numbers to thousandths, using a variety of strategies.</td>
</tr>
<tr>
<td><strong>Specific Expectations</strong></td>
</tr>
<tr>
<td>• use a variety of mental strategies to solve addition, subtract-</td>
</tr>
<tr>
<td>tion, multiplication, and division problems involving whole</td>
</tr>
<tr>
<td>numbers;</td>
</tr>
<tr>
<td>• solve problems involving the multiplication and division of</td>
</tr>
<tr>
<td>whole numbers (four-digit by two-digit), using a variety of</td>
</tr>
<tr>
<td>tools and strategies;</td>
</tr>
<tr>
<td>• multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators;</td>
</tr>
<tr>
<td>• multiply and divide decimal numbers by 10, 100, 1000, and 10 000 using mental strategies.</td>
</tr>
</tbody>
</table>

(Number Sense and Numeration, Grades 4 to 6 – Volume 4)

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)
The following sections explain content knowledge related to division concepts in the junior grades, and provide instructional strategies that help students develop an understanding of division. Teachers can facilitate this understanding by helping students to:

- interpret division situations;
- relate multiplication and division;
- use models to represent division;
- learn basic division facts;
- consider the meaning of remainders;
- develop a variety of computational strategies;
- develop estimation strategies for division.

**Interpreting Division Situations**

In the junior grades, students need to encounter problems that explore both partitive division and quotative division.

In partitive division (also called distribution or sharing division), the whole amount and the number of groups are known, but the number of items in each group is unknown.

Examples:

- Daria has 42 bite-sized granola snacks to share equally with her 6 friends. How many snacks does each friend get?
- 168 DVDs are packaged into 8 boxes. How many DVDs are there in each box?
- Zeljko’s father bought a new TV for $660. He is paying it off monthly for one year. How much does he pay each month?

In quotative division (also called measurement division), the whole amount and the number of items in each group are known, but the number of groups is unknown.

Examples:

- Thomas is packaging 72 ears of corn into bags. If each bag contains 6 ears of corn, how many bags does Thomas need?
- Anik’s class wants to raise $1100 for the Red Cross. Each month they collect $125 through fundraising. How many months will it take to raise $1100?
  (Note: In this problem, students need to deal with the remainder. For example, students might conclude that more money will need to be raised one month or that an extra month of fundraising will be needed.)
- The hardware store sells light bulbs in large boxes of 24. The last order was for 432 light bulbs. How many large boxes of light bulbs were ordered?
Students require experiences in interpreting both types of problems and in applying appropriate problem-solving strategies. It is not necessary, though, for students to identify or define these problem types.

**Relating Multiplication and Division**

Multiplication and division are inverse operations: multiplication involves combining groups of equal size to create a whole, whereas division involves separating the whole into equal groups. In problem-solving situations, students can be asked to determine the total number of items in the whole (multiplication), the number of items in each group (partitive division), or the number of groups (quotative division).

Students should experience problems such as the following, which allow them to see the connections between multiplication and division.

"Samuel needs to equally distribute 168 cans of soup to 8 shelters in the city. How many cans will each shelter get?"

"The cans come in cases of 8. How many cases will Samuel need in order to have 168 cans of soup?"

Although both problems seem to be division problems, students might solve the second one using multiplication – by recognizing that 20 cases would provide 160 cans ($20 \times 8 = 160$), and that an additional case would provide another 8 cans ($1 \times 8 = 8$), therefore determining that 21 cases would provide 168 cans. With this strategy, students, in essence, decompose 168 into ($20 \times 8$) ($1 \times 8$), and then add $20 + 1 = 21$.

Providing opportunities to solve related problems helps students develop an understanding of the part-whole relationships inherent in multiplication and division situations, and enables them to use multiplication and division interchangeably, depending on the problem situation.

**Using Models to Represent Division**

Models are concrete and pictorial representations of mathematical ideas. It is important that students have opportunities to represent division using models that they devise themselves (e.g., using counters to solve a problem involving fair sharing; drawing a diagram to represent a quotative division situation).

Students also need to develop an understanding of conventional mathematical models for division, such as arrays and open arrays. Because array models are also useful for representing multiplication, they help students to recognize the relationships between the two operations. Consider the following problem.

"In preparation for their concert in the gym, a class is arranging 72 chairs in rows of 12. How many rows will there be?"

To solve this problem, students might arrange square tiles in an array, by creating rows of 12, and discover that there are 6 rows. The array, as a model of a mathematical situation, provides
a representation of $72 \div 12 = 6$. It helps students to visualize how the factors of 12 and 6 can be combined to create a whole of 72.

Teachers can also use open arrays to help students represent division situations where it is impractical to create an array in which every square or item within the array is indicated. Consider this problem.

“The organizing committee for a play day needs to organize 112 students into teams of 8. How many teams will there be?”

Students can represent the problem using an open array.

The open array may not represent how students visualize the problem (i.e., how students will be organized into teams), and it does not provide an apparent solution to $112 \div 8$. The open array does, however, provide a tool with which students can reason their way to a solution. Students might realize that 10 teams of 8 would include 80 students but that another 32 students (the difference between 112 and 80) also need to be organized into teams of 8. By splitting the array into sections to show that 112 can be decomposed into 80 and 32, students can re-create the problem in another way.
The parts in the open array help students to determine the solution. Since \(32 \div 8 = 4\) (although many students will likely think “\(4 \times 8 = 32\)”), students can determine that the number of teams will be \(10 + 4\), or 14.

Initially, students use mathematical models, such as open arrays, to represent problem situations and their own mathematical thinking. With experience, students can also learn to use models as powerful tools to think with (Fosnot & Dolk, 2001). Appendix 4–1: Using Mathematical Models to Represent Division provides guidance to teachers on how they can help students use models as representations of mathematical situations, as representations of mathematical thinking, and as tools for learning.

**Learning Basic Division Facts**

A knowledge of basic division facts supports students in understanding division concepts and in carrying out mental computations and paper-and-pencil calculations. Because multiplication and division are related operations, students often use multiplication facts to answer corresponding division facts (e.g., \(4 \times 6 = 24\), so \(24 \div 4 = 6\)).

The use of models and thinking strategies helps students to develop knowledge of basic facts in a meaningful way. Chapter 10 in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006* (Volume 5) provides practical ideas on ways to help students learn basic division facts.

**Considering the Meaning of Remainders**

The following problem was administered to a stratified sample of 45,000 students nationwide on a National Assessment of Educational Progress secondary mathematics exam.

> “An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?”

Seventy percent of the students completed the division computation correctly. However, in response to the question “How many buses are needed?”, 29 percent of students answered “31 remainder 12”; 18 percent answered “31”; 23 percent answered “32”, the correct response (Schoenfeld, 1987).

The preceding example illustrates the impact that a mathematics program focusing on learning algorithms can have on students’ ability to interpret mathematical problems and their solutions. The example also highlights the importance of considering the meaning of remainders in division situations.

In a problem-solving approach to teaching and learning mathematics, students must consider the meaning of remainders within the context of the problem. Consider this problem.
“There are 11 players on a soccer team. 139 students signed up for an intramural soccer league. How many teams will there be?”

In solving the problem, students discover that there are 12 teams, and 7 extra players. The solution requires students to consider what can be done with the 7 additional players. Some students might distribute these players to 7 teams, whereas others might suggest smaller teams.

The following problem, which involves the same numbers as in the preceding situation but with a different context, requires students to think differently about the remainder.

“11 classmates purchased a painting for their teacher, who was moving to a new school. If the painting cost $139, how much did each classmate contribute for the gift?”

In this problem, students discover that each classmate contributes $12 but that the classmates are still short $7. Students would have to come up with a fair way to account for the shortfall.

Students can deal with remainders in division problems in several ways:

- The remainder can be discarded.
  “Alexandrea cuts 1 m of string into 30 cm pieces. How many pieces can she make?” (3 pieces, and the remaining 10 cm is discarded)

- The remainder can be partitioned into fractional pieces and distributed equally.
  “If 4 people share 5 loaves of bread, how much does each person get?” (1 and 1/4 loaves)

- The remainder can remain a quantity.
  “Six children share 125 beads. How many beads will each child get?” (20 beads, with 5 beads left over)

- The remainder can force the answer to the next highest whole number.
  “Josiah needs to package 80 cans of soup in boxes. Each box holds 12 cans. How many boxes does he need?” (7 boxes, but one box will not be full)

- The quotient can be rounded to the nearest whole number for an approximate answer.
  “Tara and her two brothers were given $25 to spend on dinner. About how much money does each person have to spend?” (about $8)

Presenting division problems in a variety of meaningful contexts encourages students to think about remainders and determine appropriate strategies for dealing with them.

**Developing a Variety of Computational Strategies**

Developing effective computational strategies for solving division problems is a goal of instruction in the junior grades. However, a premature introduction to a standard division algorithm does little to promote student understanding of the operation or of the meaning behind computational procedures. In classrooms where rote memorization of algorithmic steps is emphasized, students often make computational errors without understanding why they
are doing so. The following example illustrates an error made by a student who does not understand the division processes represented in an algorithm:

\[
\begin{array}{c|c}
81 & R 7 \\
9 & \hline
716 \\
72 & 16 \\
9 & 7 \\
\end{array}
\]

The student constructs the algorithm in his own mind as, “Come as close to the number as you can, then subtract.” Recalling multiplication facts, he knows that \(9 \times 8 = 72\) (a product that is very close to 71) and subsequently subtracts incorrectly.

**EARLY STRATEGIES FOR PARTITIVE DIVISION PROBLEMS**

Students are able to solve division problems long before they are taught procedures for doing so. When students are presented with problems in meaningful contexts, they rely on strategies that they already understand to work towards a solution. In the primary grades, students often solve partitive division problems by dealing out or distributing concrete objects one by one. When students use this strategy to divide larger numbers, they realize that dealing out objects one by one can be cumbersome, and that it is difficult to represent large numbers using concrete materials.

In the junior grades, students learn to employ more sophisticated methods of fair sharing as they develop a greater understanding of ways in which numbers can be decomposed.

“Jamie’s grandmother brought home 128 shells from her beach vacation. She wants to divide the shells equally among her 4 grandchildren. How many shells will each grandchild receive?”

To solve this problem, students might first think of 128 as 100 + 28. They realize that 100 is four 25’s and begin by allocating 25 to each of 4 groups. Students might then distribute the remaining 28 by first allocating 5 and then 2 to each group, or they might recognize that 28 is a multiple of 4 (4 \(\times 7 = 28\)) and allocate 7 to each group. After distributing 128 equally to 4 groups, students solve the problem by recognizing that each grandchild will receive 32 shells. The following illustration shows how students might represent their strategy.
The strategy of decomposing the dividend into parts (e.g., decomposing 128 into 100 + 28) and then dividing each part by the divisor is an application of the **distributive property**. According to the distributive property, division expressions, such as 128 ÷ 4, can be split into smaller parts, for example, (100 ÷ 4) + (28 ÷ 4). The sum of the **partial quotients** (25 + 7) provides the answer to the division expression.

**EARLY STRATEGIES FOR QUOTATIVE DIVISION PROBLEMS**

Division is often referred to as “repeated subtraction” (e.g., 24 ÷ 6 is the same as 24 – 6 – 6 – 6 – 6 – 6). Although this interpretation of division is correct, students in the early stages of learning division strategies often use repeated addition to solve quotative problems. For many students, it makes more sense to start at zero and add up to the dividend.

“144 baseballs are placed in trays for storage. Each tray holds 24 balls. How many trays are needed?”

To solve this problem, students might repeatedly add 24 until they get to 144, and then count the number of times they added 24 to determine the number of groups of 24, as shown at right.

Students might also use repeated subtraction in a similar way. Beginning with 144, they continually subtract 24 until they get to 0, and then count the number of times they subtracted 24.

Students demonstrate a growing understanding of multiplicative relationships when they realize that they can add or subtract “chunks” (groups of groups), rather than adding or subtracting one group at a time.

“The library just received 56 new books. The librarian wants to create take-home book packs with 4 books in each pack. How many packs can he make?”

Two methods, both involving “chunking”, are illustrated in the following strategies. In the first example (on the left), a familiar fact, 5 × 4, is used to determine that 5 packs can be created with 20 books, and therefore 10 packs can be created with 40 books. Another fact, 2 × 4, is used to determine that there are 4 packs for the remaining 16 books. In the second example (on the right), the same multiplication facts help to determine quantities that can be subtracted from 56.
It is important to note that both methods make use of the distributive property. In the first example, 56 is decomposed into \((5 \times 4) + (5 \times 4) + (2 \times 4) + (2 \times 4)\). In the second example, the number of 4s is found by decomposing 56 ÷ 4 into \((20 \div 4) + (20 \div 4) + (8 \div 4) + (8 \div 4)\). Providing opportunities for students to explore informal division strategies (which are often based on the distributive property) prepares students for understanding more formal methods and algorithms.

**DEVELOPING AN UNDERSTANDING OF THE DISTRIBUTIVE PROPERTY**

The distributive property is the basis for a variety of division strategies, including the standard algorithm. An understanding of how the property can be applied in division allows students to develop flexible and meaningful strategies, and helps bring meaning to the steps involved in algorithms.

Consider the division expression 195 ÷ 15. When instruction focuses on the algorithmic steps, students are taught to figure out how many times 15 “goes into” 19, despite the fact that 19 is really 190. A deeper understanding of the distributive property allows students to rework the problem into friendly numbers: 190 can be decomposed into 150 + 45, and each part can be divided by 15.

\[
\begin{align*}
195 &= 150 + 45 \\
150 &= 15 \times 10 \\
45 &= 15 \times 3 \\
190 &= 15 \times 13 \\
3 &= 15 \times 0 \\
10 &= 15 \times 0 \\
13 &= 15 \times 1 \\
\end{align*}
\]

Students can use an open array to model the strategy.

There is significant flexibility in using the distributive property to solve division problems. For example, the preceding division expression could have been calculated by decomposing 195 into 75 and 120, then dividing 75 ÷ 15 and 120 ÷ 15, and then adding the partial quotients \((5 + 8)\). However, strategies that use the distributive property are most effective when division expressions can be broken into friendly numbers and are easy to compute. For example, 150 ÷ 15 and 45 ÷ 15 are generally easier to compute mentally than 75 ÷ 15 and 120 ÷ 15 are.
Students learn that facts involving $10 \times$ and $100 \times$ are helpful when using the distributive property. To solve $889 \div 24$, for example, students might take a “stepped” approach to decomposing $889$ into groups of $24$.

\[
\begin{align*}
24 \times 10 &= 240 \\
24 \times 10 &= 240 \\
24 \times 10 &= 240 \\
24 \times 5 &= 120 \\
24 \times 2 &= 48 \\
&\quad 37 \quad 888
\end{align*}
\]

Students calculate that $37$ groups of $24$ is $888$, and therefore the solution is $889 \div 24 = 37 \text{ R}1$.

The strategy can be illustrated by using an open array.

\[
\begin{array}{cccccc}
10 & 10 & 10 & 5 & 2 \\
24 & 240 & 240 & 120 & 48
\end{array}
\]

When division involves large numbers, informal strategies make it difficult for students to keep track of numerical operations. In these situations, algorithms become useful to help students record and keep track of the multiple steps and operations in division.

**DEVELOPING AN UNDERSTANDING OF FLEXIBLE DIVISION ALGORITHMS**

Flexible division algorithms, like the standard algorithm, are based on the distributive property. With flexible algorithms, however, students use known multiplication facts to decompose the dividend into friendly “pieces”, and repeatedly subtract those parts from the whole until no multiples of the divisor are left. Students keep track of the pieces as they are “removed”, which is illustrated in the two examples below.

\[
\begin{align*}
17) & \quad 387 \\
- \quad 170 & \quad 10 \\
- \quad 47 & \quad 2 \\
\hline
& \quad 34 \quad 2 \\
13) & \quad 22 \\
& \quad 22 \\
\hline
& \quad 0
\end{align*}
\]

\[
\begin{align*}
26) & \quad 5562 \\
- \quad 2600 & \quad 10 \\
- \quad 260 & \quad 10 \\
- \quad 26 & \quad 2 \\
\hline
& \quad 24 \quad 1 \\
24 & \quad 213
\end{align*}
\]

\[387 \div 17 = 22 \text{ R}13\]

\[5562 \div 26 = 213 \text{ R}24\]
A student who is using a flexible algorithm to solve the first example, $387 \div 17$, might reason as follows:

"I need to divide 387 into groups of 17. How many groups can I make? I know I can get at least 10 groups. That's 170, and if I remove that, I have 217 left. Another 10 groups would leave me with 47. I can get 2 groups from that, so I can take off another 34. That leaves me with 13, which isn't enough for another group. So altogether, I made $10 + 10 + 2 = 22$ groups, and have 13 left."

As students become more comfortable multiplying and dividing by multiples of 10, they learn to compute using fewer partial quotients in the algorithm, as illustrated below:

![Division Algorithm Example](image)

**DEVELOPING AN UNDERSTANDING OF THE STANDARD DIVISION ALGORITHM**

Historically the algorithms (standardized steps for calculation) were created to be used for efficiency by a small group of “human calculators” when calculators were not yet invented. They were not designed to support the sense making that is now expected from students. (Teaching and Learning Mathematics in Grades 4 to 6 in Ontario, 2004, p. 12)

Although the standard division algorithm provides an efficient computational method, the steps in the algorithm can be very confusing for students if they have not had opportunities to solve division problems using their own strategies and methods.

Working with flexible division algorithms can prepare students for understanding the standard algorithm. A version of the flexible division algorithm involves stacking the quotients above the algorithm (rather than down the side, as demonstrated in the above example). The following example shows how the parts in the flexible algorithm can be connected to the recording method used in the standard algorithm.
Developing Estimation Strategies for Division

Students need to develop effective estimation strategies, and they also need to be aware of when one strategy is more appropriate than another. It is important for students to consider the context of a problem before selecting an estimation strategy. Students should also decide beforehand how accurate their estimation needs to be. Consider the following problem.

"Ms. Wu’s class is putting cans in boxes for the annual canned-food drive. They have 188 cans and put approximately 20 cans in a box. About how many boxes do they need?"

In this problem situation, it is useful to use an estimation strategy that results in enough boxes to package all the cans (e.g., round 188 to 200 and divide by 20 to get 10 boxes).

The following table outlines different estimation strategies for division. It is important to note that the word “rounding” is used loosely – it does not refer to any set of rules or procedures for rounding numbers (e.g., look to the number on the right; is it greater than 5? …).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounding the dividend and/or divisor to the nearest multiple of 10, 100, 1000, …</td>
<td>442 ÷ 50 is about 450 ÷ 50 = 9 785 ÷ 71 is about 800 ÷ 80 = 10</td>
</tr>
<tr>
<td>Finding friendly numbers</td>
<td>318 ÷ 25 is about 325 ÷ 25 = 13</td>
</tr>
<tr>
<td>Rounding the dividend up or down and adjusting the divisor accordingly</td>
<td>237 ÷ 11 is about 240 ÷ 12 = 20 237 ÷ 11 is about 230 ÷ 10 = 23</td>
</tr>
<tr>
<td>Using front-end estimation (Note that this strategy is less accurate with division than with addition and subtraction.)</td>
<td>453 ÷ 27 is about 400 ÷ 20 = 20 (actual answer is 16 R21)</td>
</tr>
<tr>
<td>Finding a range (by rounding both numbers down, then up)</td>
<td>565 ÷ 24 is about 500 ÷ 20 = 25 565 ÷ 24 is about 600 ÷ 30 = 20 The quotient is between 20 and 25.</td>
</tr>
</tbody>
</table>
A Summary of General Instructional Strategies

Students in the junior grades benefit from the following instructional strategies:

• experiencing a variety of division problems, including partitive and quotative problems;
• using concrete and pictorial models to represent mathematical situations, to represent mathematical thinking, and to use as tools for new learning;
• solving division problems that serve different instructional purposes (e.g., to introduce new concepts, to learn a particular strategy, to consolidate ideas);
• providing opportunities to develop and practise mental computation and estimation strategies;
• providing opportunities to connect division to multiplication through problem solving.

The Grades 4–6 Multiplication and Division module at www.eworkshop.on.ca provides additional information on developing division concepts with students. The module also contains a variety of learning activities and teaching resources.
APPENDIX 4-1: USING MATHEMATICAL MODELS TO REPRESENT DIVISION

The Importance of Mathematical Models
Models are concrete and pictorial representations of mathematical ideas, and their use is critical in order for students to make sense of mathematics. At an early age, students use models such as counters to represent objects and tally marks to keep a running count. Standard mathematical models, such as number lines and arrays, have been developed over time and are useful as "pictures" of generalized ideas. In the junior grades, it is important for teachers to develop students' understanding of a variety of models so that models can be used as tools for learning.

The development in understanding a mathematical model follows a three-phase continuum:

- **Using a model to represent a mathematical situation:** Students use a model to represent a mathematical problem. The model provides a "picture" of the situation.
- **Using a model to represent student thinking:** After students have discussed a mathematical idea, the teacher presents a model that represents students' thinking.
- **Using a model as a tool for new learning:** Students have a strong understanding of the model and are able to apply it in new learning situations.

An understanding of mathematical models takes time to develop. A teacher may be able to take his or her class through only the first or second phase of a particular model over the course of a school year. In other cases, students may quickly come to understand how the model can be used to represent mathematical situations, and a teacher may be able to take a model to the third phase with his or her class.

**USING A MODEL TO REPRESENT A MATHEMATICAL SITUATION**
A well-crafted problem can lead students to use a mathematical model that the teacher would like to highlight. The following example illustrates how the use of an array as a model for division might be introduced.
A teacher provides students with the following problem:

"Students in the primary division are putting on a concert, and the principal has asked our class to set up chairs in the gym for parents and guests. We have 345 chairs, and the principal wants rows with 15 chairs in each row. How many rows do we need to set up?"

The teacher has purposefully selected the numbers in the problem: they are friendly (easy to work with) but large enough to prevent quick solutions. They are also too large for students to use counters or other manipulatives, and repeated subtraction or repeated addition would be inefficient strategies. To this point, the class has not been taught any formal algorithms for division by a two-digit number.

The problem also lends itself to the use of an array. Although some students attempt to solve the problem using only numerical calculations, others use drawings to recreate the situation. One student uses grid paper, with each square representing a chair:

The student explains her strategy:

"I started drawing rows of 15 chairs. I knew that 10 rows would have 150 chairs, because I know that $15 \times 10 = 150$. So I drew a line around 10 rows, and wrote 150. Another 10 rows would give me another 150, for a total of 300 chairs. That just left 45 chairs, which is 3 rows of 15. So I know we’d have 10 rows plus 10 rows plus 3 rows, for a total of 23 rows."

This student used an array to model the rows of chairs. Although not all students used this model, the teacher is able to draw attention to it during the Reflecting and Connecting part of the lesson. The student, having no formal strategy for dividing by two-digit numbers, has used an array to represent 345 chairs in rows of 15, and then has broken the array into parts to determine the total number of rows.
The student has also used another important idea in division – making groups of tens. Not all students used the array as a model to represent the mathematical situation, and there is no guarantee that students who did use it can or will apply it to other division problems. It is the teacher’s role to help students generalize the use of the array as a model in other division situations.

**USING A MODEL TO REPRESENT STUDENT THINKING**

Teachers can guide students in recognizing how models can represent mathematical thinking. The following example provides an illustration.

After solving problems in which the class used arrays to represent division situations, a teacher presents the following problem:

“My neighbour is a potter and is well known for her unique coffee mugs. She sells them to kitchen stores in sets of 12, in special boxes that protect the mugs during shipping. Yesterday, a store placed an order for 288 mugs. She needs to know how many boxes she needs to ship the mugs to the store.”

The teacher encourages students to use strategies that make sense to them, and suggests that they use concrete materials and diagrams to help them understand and solve the problem. One student solved the problem mentally, recording the results of his mental calculations on paper as he worked through the problem.

The teacher, wanting to highlight the student’s strategy with the class, asks the student to explain his work. The student explains:

“I figured out that the problem is finding how many groups of 12 there are in 288. I started thinking about numbers I knew. I knew 10 groups would be 120 mugs, and another 10 is 240. I subtracted 240 from 288 and had 48 left. That’s 4 more groups of 12, so in total I had (20 + 4) 24 groups. I checked by multiplying 24 × 12 and got 288.”

---

The student has also used another important idea in division – making groups of tens.

Appendix 4–1: Using Mathematical Models to Represent Division

<table>
<thead>
<tr>
<th>288 – 12 … how many groups of 12?</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 groups is 120</td>
</tr>
<tr>
<td>20 groups is 240</td>
</tr>
<tr>
<td>288 – 240 = 48</td>
</tr>
<tr>
<td>48 ÷ 12 = 4</td>
</tr>
<tr>
<td>So, 20 groups + 4 groups is 24 groups</td>
</tr>
<tr>
<td>Check: 24</td>
</tr>
<tr>
<td>× 12</td>
</tr>
<tr>
<td>48</td>
</tr>
<tr>
<td>240</td>
</tr>
<tr>
<td>288</td>
</tr>
</tbody>
</table>

"I figured out that the problem is finding how many groups of 12 there are in 288. I started thinking about numbers I knew. I knew 10 groups would be 120 mugs, and another 10 is 240. I subtracted 240 from 288 and had 48 left. That’s 4 more groups of 12, so in total I had (20 + 4) 24 groups. I checked by multiplying 24 × 12 and got 288."
Although the student did not use an array model to solve the problem, his teacher presents an open array to the class to help students visualize their classmate’s thinking. The solution is represented through a series of diagrams.

“If we think about the problem as an array, then the area of the array is 288, and the length of one side is 12. We need to find the length of the other side. You solved the problem by breaking 288 into friendlier numbers: $120 + 120 + 48$.”

In this case, the array is used to model a strategy in which partial quotients are determined by using friendly numbers that are multiples of 10. The dividend has been decomposed into numbers that are easier to work with.

The teacher has provided a visual representation of a student solution that makes the strategy more accessible to other students in the class, and has built upon students’ understanding of the array model. With meaningful practice rooted in contextual problems, the open array model can become a useful tool for dividing numbers.

**USING A MODEL AS A TOOL FOR NEW LEARNING**

To help students generalize the use of an open array as a model for division, and to help them recognize its utility as a tool for learning, teachers need to provide problems that allow students to apply and extend the strategy of partial quotients. A sample problem comes out of a fundraising scenario.

“23 students raised $437 for the United Way. If each student brought in the same amount of money, how much did each student raise?”

The numbers in the problem were chosen to be challenging, but they also allow for various strategies to find a solution. Many students use a strategy that involves determining partial quotients by decomposing 437. To begin, they recognize that $10 \times 23 = 230$ and draw an open array to represent this idea. They continue to multiply 23 by other factors, drawing other sections on their diagram until 437 has been accounted for.
"I kept multiplying 23 by friendly numbers. I started with 10, and got 230. I tried another 10, but that would have given me 460, which is too much. So I timesed by 5, which was easy because it was half of 10. I kept going that way, trying numbers that fit. Each time I tried a new number, I had to take it away from 437 to find out how much was left. It ended up that each student raised $19."

Another student started down a similar path but used the distributive property and subtraction instead of addition.

"I knew that 20 times 23 is 460, which was more than I needed. I subtracted 437 from 460, and found the difference was 23. So that's 1 group of 23 less, or 19 groups. So 437÷23 = 19."

In both cases, students used an open array as a tool for solving a division problem. The first student used a strategy that replicates a solution modelled by the teacher, but the second
student used the model to come up with a compensation strategy. The second student used the open array model as a tool for solving a division problem in a new way.

When developing a model for division, it is practical to assume that not all students will come to understand or use the model with the same degree of effectiveness. Teachers should continue to develop meaningful problems that allow students to use strategies that make sense to them. However, part of the teacher’s role is to use models to represent students’ ideas so that these models will eventually become thinking tools for students. The ability to generalize a model and use it as a learning tool takes time (possibly years) to develop.
REFERENCES


Learning Activities for Division

Introduction
The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to division. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or activity.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students’ understanding of mathematical concepts.
HOME CONNECTION: This section is addressed to parents or guardians, and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.
Grade 4 Learning Activity
Intramural Dilemmas

OVERVIEW
In this learning activity, students solve problems by dividing groups of students into teams of equal size. The focus in this activity is on solving different kinds of division problems (e.g., partitive, quotative) and on having students use meaningful strategies. Allowing students to develop and apply their own strategies helps them develop an understanding of division situations, and of flexible approaches for solving division problems.

BIG IDEAS
This learning activity focuses on the following big ideas:

Operational sense: Students solve division problems by using strategies that make sense to them. They discover that solutions to division problems sometimes involve a remainder and that the remainder must be dealt with within the context of the problem.

Relationships: By solving division problems, students explore the relationship involving a quantity, the number of groups the quantity can be divided into, and the size of each group. They also explore the relationships between the operations, particularly between division and repeated subtraction.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectation.

Students will:
• divide two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., concrete materials, drawings) and student-generated algorithms.

This specific expectation contributes to the development of the following overall expectation.

Students will:
• solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.
ABOUT THE LEARNING ACTIVITY

MATERIALS
• a variety of manipulatives (e.g., counters, base ten blocks, square tiles)
• sheets of chart paper or large sheets of newsprint (1 per pair of students)
• markers (a few per pair of students)
• Div4.BLM1: Intramural Dilemmas (1 per student)
• Div4.BLM2: The Remainders Game (1 per student)

MATH LANGUAGE
• division
• quotient
• divide
• divisible
• dividend
• remainder
• divisor

INSTRUCTIONAL SEQUENCING
This learning activity provides an introductory exploration of strategies for dividing a two-digit number by a one-digit number. Before starting this learning activity, students should have had experiences in dividing quantities into equal-sized groups.

ABOUT THE MATH
Students develop an understanding of division concepts when they solve problems that involve separating a quantity into equal groups. Partitive division involves situations in which the quantity is separated into a specified number of equivalent groups. The quotient indicates the number of items in each group.

EXAMPLE OF A PARTITIVE DIVISION PROBLEM
A grocer has 30 apples. He puts the apples into 5 bags so that each bag contains the same number of apples. How many apples will the grocer put in each bag?

Quotative division involves situations in which a quantity is separated into groups of a specified size. The quotient refers to the number of groups that can be formed.

EXAMPLE OF A QUOTATIVE DIVISION PROBLEM
A grocer has 30 apples. She wants to put 5 apples in each bag. How many bags will the grocer need?

It is not important that students be able to identify division problems as either partitive or quotative. It is important, however, to provide opportunities for students to solve both kinds of problems and deepen their understanding of division concepts.
It is also essential to allow students to solve problems in ways that make sense to them, so that they can construct a meaningful understanding of division and its relationship to other operations. Students’ solution strategies vary in sophistication – some students use simple counting strategies, while others apply their understanding of number strategies, such as using repeated subtraction, using multiplying as the inverse of dividing, or using basic division facts.

GETTING STARTED

Explain to students that it is often necessary to divide a class into equal-sized groups for games, sports, and learning activities. Display the following problems on the board or on chart paper, and discuss them with students:

- We want to play checkers. How many groups will there be if we divide our class into groups of 2?
- We want to play soccer. How many students will there be on each team if we divide our class into 2 equal groups?

Ask students to work with a partner to solve each problem by using a strategy that makes sense to them. Suggest that students use manipulatives (e.g., counters, base ten blocks, square tiles) and/or diagrams to help them determine a solution.

Provide each pair of students with markers and a sheet of chart paper or large sheet of newsprint. Ask them to fold the paper in half and record a solution for each problem on each part of the paper. Ask each pair to be prepared to share their strategies and solutions with their classmates.

Watch and listen to students as they work on the problems, observe the various strategies being used, and provide guidance when necessary. If some students finish before others, encourage them to find other ways to solve the problems.

Note: If there is an odd number of students in the class, students will need to discuss how the extra person can be included in the groups (e.g., forming one group of three for the checker games or having a soccer team with an extra player).

STRATEGIES STUDENTS MIGHT USE

COUNTING

Students might use manipulatives (e.g., counters, base ten blocks, square tiles) to represent the students in the class. To solve the first problem, they might count out the number of manipulatives that corresponds to the number of students in the class, then arrange the manipulatives into groups of two, and then count the number of groups. For the second problem, students could divvy the manipulatives into two equal groups and count the number of manipulatives in each group.
USING REPEATED SUBTRACTION

Students might use repeated subtraction to solve the first problem. If there are 26 students in the class, they might subtract 2 from 26, and then continue to subtract 2 from the remaining difference. Finally, students would determine that there were 13 groups of two by counting the number of times they subtracted 2.

The second problem does not lend itself to a process of repeated subtraction. Instead, students might use a halving process (e.g., recognizing that one half of 26 is 13).

DECOMPOSING A NUMBER INTO PARTS

Students can determine the number of groups in the first problem by decomposing the number of students into tens and ones. For example, if there are 26 students, they might break 26 into 2 tens and 6 ones. There are 5 pairs in each group of 10 students and 3 pairs in the group of 6 students.

10 students → 5 pairs
10 students → 5 pairs
6 students → 3 pairs
13 pairs altogether

Gather students together after they have had sufficient time to solve the problems. Select a few pairs to present and discuss their solutions, choosing students who used different strategies.

During students’ presentations, avoid making comments that suggest that some strategies are better than others. Instead, encourage students to consider the effectiveness and efficiency of each strategy by asking the following questions after each presentation:

• "Was it easy to find a solution using your strategy?"
• "What worked well?"
• "What did not work well?"
• "How would you change your strategy if you solved the problem again?"

WORKING ON IT

Explain to students that their help is needed in solving a problem about organizing intramural teams. Present the context for the problem:

“Seventy-eight students signed up for intramural sports. All the students will play both soccer and four-square. Ms. Boswell [Note: You might want to use the name of a teacher in your school who is involved in organizing intramural teams] would like to create the different teams and then make a chart with the names for each team. The chart will allow students to see which teams they belong to. But first of all, Ms. Boswell needs to solve some problems.”
Present the following problems on the board or on chart paper:
• There will be 4 soccer teams. How many players will there be on each team?
• There are 4 players on each four-square team. How many teams will there be?

Ask students to work with a partner. (You might decide to have students work with the same partner as in Getting Started, or you might form different pairs.) Explain that students may solve the problems in any order.

Encourage students to consider whether any of the various strategies that were demonstrated earlier could help them solve the problems. Encourage them, as well, to modify any of the strategies or to develop new ones. Ensure that manipulatives (e.g., counters, base ten blocks, square tiles) are available, and invite students to use them.

Provide each student with a copy of Div4.BLM1: Intramural Dilemmas. Explain that although they are working in pairs, each student is responsible for recording solutions to the problems. Remind students to think about ways to use words, symbols, and/or diagrams to explain their ideas.

REFLECTING AND CONNECTING

After students have solved the problems, use an inside-outside circle strategy (described below) so that they can share their solutions with each other. Have students use their completed copy of Div4.BLM1: Intramural Dilemmas, and organize them for the activity:

• Divide the class into two equal groups.
• Ask one group to form a circle with students facing outwards, away from the centre of the circle.
• Ask the other group to form another circle around the first circle. Each student in the inside circle faces a partner in the outside circle.

To begin the activity, ask students to discuss with their partner how they determined the number of players on each soccer team. Encourage students to refer to their work on Div4.BLM1: Intramural Dilemmas as they explain their strategies. Remind students to be courteous by allowing time for their partners to present their ideas, by listening attentively, and by making positive comments (e.g., “I think you had a clever idea!”). Allow three to four minutes for students to share their strategies for this problem.

Next, ask the outside circle to move counterclockwise by three people, and have students share their strategies and solutions for the soccer teams problem with their new partners.

Ask the inside circle to move counterclockwise by four people. Have students, with their new partners, discuss the strategies they used to determine the number of four-square teams. Again, allow three to four minutes for students to share their strategies.

Conduct another rotation to provide an opportunity for students to share their strategies and solutions for the four-square teams problem with another partner.

Following the inside-outside circle activity, discuss the problems, one at a time, with the entire group by asking the following questions.
QUESTIONS FOR THE SOCCER TEAMS PROBLEM
- “How many players will there be on each soccer team?”
- “Was there a remainder (a leftover quantity) when you divided 78 by 4? What does this leftover quantity represent? How did your solution include these 2 extra students?”
- “Is it reasonable to have 19 (or 20, if the leftover students are placed on teams) players on each team?” (Students might suggest that a team of 19 players is too large and that some players would have little opportunity to play.)
- “If a team of 19 players is too large, what would you recommend to the teacher?”

QUESTIONS FOR THE FOUR-SQUARE TEAMS PROBLEM
- “How many four-square teams will there be?”
- “Was there a remainder when you solved the problem? What does this leftover quantity represent? How did your solution include these 2 extra students?”
- “Do you think that it is appropriate to have 19 four-square teams? Why or why not?”

Ask a few students to explain their strategies to the entire group. Select different strategies, and emphasize the idea that a variety of strategies are possible. Encourage students to think about the efficiency of different strategies by asking the following questions:
- “Which strategies work well? Why?”
- “Which strategy makes the most sense to you? Why?”
- “Which strategies are similar? How are they alike?”
- “How could you change a strategy to make it more efficient?”

Next, focus on the relationships between the problems and between their solutions. Ask: “Which problems are similar?” Students might observe that the problems are alike because 4 is the divisor in both situations. Discuss the differences between the problems (e.g., in the soccer teams problem, the number of teams was known, but the size of each team was unknown; in the four-square teams problem, the size of each team was known, but the number of teams was unknown).

ADAPTATIONS/EXTENSIONS
Some students may need to work with smaller numbers. Rather than dealing with problems that involve 78 students, they could determine how 24 students could be arranged on teams. Encourage these students to use manipulatives (e.g., counters, cubes, square tiles) to represent and model the problems.

Extend the activity for students requiring a greater challenge by asking them to determine different ways in which 78 students could be divided exactly into equal teams (i.e., with no remainders).

ASSESSMENT
Observe students while they are solving the problems. Assess how well they are able to determine and apply a strategy that allows them to solve the problems effectively. Ask questions such as the following:
- “How are you solving these problems?”
- “What is working well with your strategy?”
• “What is not working well?”
• “Did you change your strategy? If yes, how?”
• “How are the problems the same?”
• “How are the problems different?”
• “How are you recording your solution?”
• “How do you know that your solution will be understood by others?”
• “Did solving one problem help you solve another? If yes, how?”

Provide the following problems for students to solve individually:

• Suppose we need to divide our class into 3 groups for science activities. How many students
  would there be in each group?
• Suppose we need to divide our class into groups of 3 for a rock-paper-scissors challenge.
  How many groups would there be?

Observe completed solutions to assess how well students:

• apply an appropriate strategy to solve the problems;
• use an efficient strategy;
• explain their strategies;
• recognize and apply the relationship between the problems.

HOME CONNECTION

Send home Div4.BLM2: The Remainders Game. This Home Connection activity provides
an opportunity for students to observe that reminders often occur when a set of items is
divided into equal groups.

LEARNING CONNECTION 1

Divide and Draw

MATERIALS

• base ten blocks, including ones cubes and tens rods (10 small cubes and 10 rods for each
  pair of students)
• paper bag labelled “ones”, containing number cards for 1 to 9 (1 per pair of students)
• paper bag labelled “tens”, containing number cards for 1 to 9 (1 per pair of students)
• six-sided number cubes (1 per pair of students)
• Div4.BLM3: Divide and Draw (1 per student)

Arrange students into pairs. Explain the activity:

• The first student draws a number card from both the “tens” and the “ones” bags, and
  selects the corresponding number of tens rods and ones cubes.
• The second student rolls the number cube to determine the number of groups into which
  the base ten blocks are to be divided.
• Students work together to divide the blocks into equal groups, exchanging tens rods for
  ones cubes when necessary.
After students have divided the blocks into groups, they record the results on Div4.BLM3: Divide and Draw using diagrams and symbols. (Students can use blank sheets of paper if they complete all sections of Div4.BLM3: Divide and Draw.)

Show students how they might record 43 divided by 3.

As students participate in the activity, ask them to explain their strategies for dividing the base ten blocks. Observe whether students divide tens before ones, or vice versa. (Both methods are acceptable.)

You can modify the activity by having students work to solve quotative problems. As before, students draw "tens" and "ones" cards, and represent the number using base ten blocks. In this modified activity, the number cube indicates the number in each group, and students need to divide the blocks to determine the number of groups.

**LEARNING CONNECTION 2**

**Decisions, Decisions**

**MATERIALS**

- six-sided number cubes (3 per pair of students)
- variety of manipulatives (e.g., counters, square tiles, base ten blocks)

Have students play this game with a partner. Explain that the goal of the game is to be the player who creates the division sentence with the greater quotient.

For each round of the game, both players record the following empty division expression on a sheet of paper.

\[
\underline{\text{______}} \div \underline{______}
\]
To begin, the first player rolls three number cubes and records each number in one of the blanks in the empty division expression. Once a number is recorded, it cannot be moved. The second player follows the same procedure to create a division expression on his or her paper.

Next, players determine the solutions to their division expressions using any strategy (e.g., using manipulatives, using basic facts, drawing a diagram, using paper-and-pencil calculations), and record the answer on their paper.

**Example:**
Player A rolls 6, 3, and 5 and completes the division expression in the following way:

\[
\begin{array}{c}
5 \\
6 \\
\div 3
\end{array}
= 18 R2
\]

Player B rolls 4, 4, and 6 and completes the division expression in the following way:

\[
\begin{array}{c}
6 \\
4 \\
\div 4
\end{array}
= 16
\]

Players check each other’s work. Players earn 2 points if they get the correct answer. As well, the player with the greater quotient earns 5 points.

The player with the greater quotient begins the next round.

The first player to earn 50 points wins the game.

After students have played the game, discuss the strategies they used to determine the greatest possible quotient.

Variations of the game can involve:
* using a 10-sided number cube with numbers 0 to 9;
* trying to create the lesser quotient;
* trying to create the greater remainder;
* trying to create the lesser remainder.

**LEARNING CONNECTION 3**

**Fair Shares**

**MATERIALS**
* paper bag containing between 20 and 40 square tiles (1 bag per group of 4 students)

Divide students into groups of four. Provide each group with a bag of square tiles. Ask students to pour the tiles onto their desks and to count the tiles. Have them print this number on the bag.

Next, ask students to discuss the following question with their group members: “Can you share the tiles fairly between 2 of the group members and have no remainders?” After students have discussed the question, invite them to check their prediction by divvying the tiles between 2 students.

Next, ask: “Can you share the tiles fairly among 3 of the group members and have no remainders?” Have students discuss the question with group members before checking their predictions.
Finally, ask students to discuss whether there will be a remainder if the tiles are shared among 4 group members, and have them verify their conjectures.

Have groups exchange bags and repeat the activity.

As a whole class, discuss the following questions:
• “How did you predict whether or not there would be a remainder?”
• “When was it easy to predict that there would be no remainder? Why?”
• “Which number of tiles could be shared the greatest or the least number of ways? Why do you think this?”

As an extension, ask students to predict whether remainders would occur if the tiles in their bag were shared among 5 students, 6 students, and 7 students.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on division concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Multiplication and Division (4 to 6)”, and then click the number to the right of it.
Intramural Dilemmas

There are 78 students who signed up for intramural sports.

There are 4 soccer teams. How many players are there on each team?

There are 4 players on each four-square team. How many teams are there?
Dear Parent/Guardian:

We have been learning to solve division problems. Sometimes there are remainders in a division problem. If you take 23 buttons and divide them into 4 equal groups, there will be 5 buttons in each group and 3 leftover buttons.

Play the Remainders Game with your child, to help him or her practise making groups with equal amounts and determining whether there are any remainders.

Thank you for doing this activity with your child.

THE REMAINDERS GAME

Play this game with a partner. You will need a number cube (die) and 20 small objects (e.g., buttons, paper clips, pennies, pieces of paper).

To begin, a player rolls the number cube. The player needs to divide the 20 small objects into the number of groups shown on the number cube. For example, if a player rolls a 3, the player divides the 20 small objects into 3 groups.

If there is a remainder (there would be 2 leftover objects in the previous example), the player records the number of remaining objects on a piece of paper.

Now the turn passes to the other player who rolls the number cube, makes equal groups, and records the remainder on his or her paper.

After each player has had 10 turns, the players add up the remainders on their piece of paper. The player with the greater total wins the game.
Divide and Draw

_____ divided into _____ groups

_____ divided into _____ groups

_____ divided into _____ groups
Grade 5 Learning Activity
Family Math Night

OVERVIEW
In this learning activity, students solve a problem by determining the number of tables that are needed for 165 people if 6 people sit at each table. The focus in this learning activity is on having students use strategies that make sense to them, rather than on applying learned procedures.

BIG IDEAS
This learning activity focuses on the following big ideas:

Operational sense: Allowing students to develop and apply their own strategies helps them develop an understanding of division situations and of flexible approaches for solving division problems.

Relationships: The learning activity provides an opportunity for students to explore the relationship involving a quantity, the number of groups the quantity can be divided into, and the size of each group.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations.
Students will:
• divide three-digit whole numbers by one-digit whole numbers, using concrete materials, estimation, student-generated algorithms, and standard algorithms;
• use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution.

These specific expectations contribute to the development of the following overall expectation.
Students will:
• solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.
ABOUT THE LEARNING ACTIVITY

MATERIALS
• sheets of paper (1 per pair of students)
• pencils
• sheets of chart paper or large sheets of newsprint (1 per pair of students)
• markers (a few per pair of students)
• sheets of paper or math journals (1 per student)
• Div5.BLM1: Sharing Pennies (1 per student)

MATH LANGUAGE
• divide • remainder
• division • divisible
• divisor • algorithm
• quotient • multiples

INSTRUCTIONAL SEQUENCING
This learning activity provides an introductory exploration of strategies for dividing a three-digit number by a one-digit number. Before starting this learning activity, students should have had experiences in solving problems involving the division of two-digit numbers by one-digit numbers.

ABOUT THE MATH
In Grade 5, students are expected to divide three-digit whole numbers by a one-digit divisor. Although students are working with larger numbers than in previous grades, instructional activities should continue to focus on the meaning of division, not merely on teaching paper-and-pencil algorithms.

In this learning activity, students solve a problem by using strategies that make sense to them. When students design their own strategies, they need to interpret the problem situation and apply their understanding of number operations, rather than simply following the steps in an algorithm. Given this opportunity to solve division problems in ways that make sense to them, students use a variety of strategies of varying complexity and efficiency. At the conclusion of the learning activity, students discuss the efficiency of different strategies for division.

GETTING STARTED
Explain to students that their help is needed in organizing a Family Math Fair at the school. Tell them that the principal has conducted a survey and that 165 people from the community have indicated that they will attend the fair. Explain to students that the first part of the math night involves a presentation in the gym and that people will sit at tables in groups of 6. Ask: “How many tables need to be set up?”
Record important information about the problem on the board:

- 165 people
- 6 people at each table
- How many tables?

Ask students to estimate the number of tables that will be needed. Discuss the different strategies that students used.

Organize the class into pairs. Ask students to work collaboratively to solve the problem in a way that makes sense to both partners. Provide each pair with a sheet of paper on which students can record their work.

WORKING ON IT

As students work on the problem, observe the various strategies they use to solve it. Pose questions to help students think about their strategies and solutions:

- "What strategy are you using to determine the number of tables that will be needed?"
- "Why did you choose this method?"
- "What is working well? What is not working well?"
- "Did you change your strategy? Why did you change it?"
- "How are you recording your ideas?"

STRATEGIES STUDENTS MIGHT USE

COUNTING
Students might draw diagrams (e.g., make tally marks) to represent 165 people, and then group the people (e.g., by circling tally marks) into sets of 6. Students then count the number of sets to determine the number of tables that are needed. Students will find that there are 27 groups of 6 people with 3 people left over and conclude that another table will be needed.

USING REPEATED ADDITION
Students might draw tables (e.g., rectangles) and indicate 6 people at each table (e.g., by sketching 6 chairs at each table, by recording "6" at each table). As they draw the tables, students keep a running count of the number of people by repeatedly adding 6 until they reach 162. Students might realize that 168, the next multiple of 6, is greater than 165, but that an extra table will be needed to accommodate the last 3 people. Students then count the number of groups of 6. Students might also use repeated addition without the use of a diagram. For example, they might repeatedly add 6, and then count the number of 6's that were added together.

USING PROPORTIONAL REASONING
Students might use proportional reasoning, for example, a doubling strategy – 1 table for 6 people, 2 tables for 12 people, 4 tables for 24 people, and so on. Students might organize this information in a table.
If students use a doubling strategy, they will observe that 16 tables are too few and that 32 tables are too many. They might combine different table-people ratios to determine the total number of tables needed; for example, 16 + 8 + 4 tables (28 tables) will seat 96 + 48 + 24 people (168 people).

**USING REPEATED SUBTRACTION**

Students might begin with 165 and repeatedly subtract groups of 6 until they reach 3. To determine the number of tables, students count the number of times that 6 was subtracted and include an extra table for the remaining 3 people.

**USING “CHUNKING”**

Students might subtract “chunks” (multiples of 6) from 165.

\[
\begin{align*}
165 & \quad - 60 \quad (10 \text{ tables}) \\
105 & \quad - 60 \quad (10 \text{ tables}) \\
45 & \quad - 42 \quad (7 \text{ tables}) \\
3 & \quad \phantom{0}
\end{align*}
\]

**USING PARTIAL QUOTIENTS**

Students might use a strategy in which they calculate partial quotients by using their knowledge of multiplication. For example, they might know that 20 tables would seat 120 people (20 \times 6 = 120), and then determine that another 8 tables (8 \times 6 = 48) would be needed for the remaining 45 people. The partial quotients, 20 and 8, are added to determine the number of tables.

**USING AN ALGORITHM**

Students might have been taught an algorithm and apply these procedures to solve the problem. If students are unable explain the meaning of the procedures and numbers in the algorithm, suggest that they select a method that they can explain.

When students have solved the problem, provide each pair with markers and a sheet of chart paper or a large sheet of newsprint. Ask students to record their strategies and solutions on the paper and to clearly show how they solved the problem.

Make a note of pairs who might share their strategies and solutions during Reflecting and Connecting. Aim to include pairs who used various methods that range in their degree of efficiency (e.g., using counting, using repeated addition, using proportional reasoning, using partial quotients).
REFLECTING AND CONNECTING

Gather the class. Ask a few pairs to share their problem-solving strategies and solutions. Try to order the presentations so that students observe inefficient strategies (e.g., counting, using repeated addition) first, followed by increasingly efficient methods. Post students’ work following each presentation.

Avoid making comments that suggest that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

As students explain their work, ask questions that encourage them to explain the reasoning behind their strategies:

- “How did you determine the number of tables that are needed?”
- “Why did you use this strategy?”
- “What worked well with this strategy? What did not work well?”
- “How do you know that your solution makes sense?”

Following the presentations, encourage students to consider the effectiveness and efficiency of the various strategies that have been presented. Ask the following questions:

- “In your opinion, which strategy worked well?”
- “Why is the strategy effective in solving this kind of problem?”
- “How would you explain this strategy to someone who has never used it?”

Provide an opportunity for students to extend their understanding of division strategies by posing the following problem:

“After the presentation in the gym, the 165 math fair participants will be divided into teams of 4 people to play math games. How many teams will there be?”

Have students work independently to solve the problem. Encourage them to think back to the different strategies presented by their classmates, and to use an efficient strategy that makes sense to them. Have students show their strategies and solutions on a sheet of paper or in their math journals.

ADAPTATIONS/EXTENSIONS

Encourage students to solve the problem by using a strategy that makes sense to them. Recognize that some students may need to use simple strategies (e.g., counting, using repeated addition, using repeated subtraction). It may be necessary to model the use of manipulatives and simple counting strategies for students who experience difficulty in solving the problem. These students might also benefit from working with a partner who is able to explain different strategies.

For students requiring a greater challenge, have them solve the problem in different ways, and ask them to explain how the various strategies are alike and different.

The following problem could also be used as an extension to the learning activity:

“78 children and 87 adults are planning to attend the math fair. Each child will receive 2 glasses of juice, and each adult will receive 1 glass of juice. A jug of juice holds 7 glasses. How many jugs of juice will be needed?”
ASSESSMENT
Observe students as they solve the problem to assess how well they:
• understand the problem;
• apply an appropriate problem-solving strategy;
• judge the efficiency and accuracy of their strategy;
• find and explain a solution;
• determine whether the solution is reasonable;
• explain their strategies and solutions clearly and concisely, using mathematical language.

Collect students’ solutions to the problem in which they determined the number of math teams of 4 people. Observe the work to determine how well they apply an efficient strategy to solve the division problem.

HOME CONNECTION
Send home Div5.BLM1: Sharing Pennies. This Home Connection activity provides an opportunity for parents and students to discuss division strategies.

LEARNING CONNECTION 1
Apples at the Math Fair

MATERIALS
• sheets of paper (1 per pair of students)

Arrange students in pairs. Ask students to solve the following problem with their partner and to record their strategy and solution on a sheet of paper.

“The principal purchased 12 dozen apples for the math fair. The apples are placed on trays that hold 8 apples each. How many trays of apples are there?”

After students have solved the problem, have each pair partner with another pair to create groups of four. Have students compare the strategies they used to solve the problem.

LEARNING CONNECTION 2
Exploring Divisibility

MATERIALS
• Div5.BLM2: Hundreds Chart (1 per student)
• pencil crayons

Recognizing divisibility (i.e., knowing that numbers divide evenly without a remainder) is important in developing mental division skills.
Provide each student with a copy of Div5.BLM2: Hundreds Chart, and ask them to use a red pencil crayon to circle the multiples of 2. Ask students to describe numerical patterns that they notice (e.g., all multiples of 2 are even numbers; multiples of 2 end in 0, 2, 4, 6, or 8).

Next, have students use a green pencil crayon to circle the multiples of 3. Again, discuss numerical patterns (e.g., the sum of the digits in a multiple of 3 is divisible by 3).

Continue by having students use a yellow pencil crayon to circle multiples of 5. Discuss numerical patterns (e.g., multiples of 5 end in 0 and 5).

Challenge students to suggest numbers greater than 100 that are divisible by 2, 3, or 5, and ask them to explain why the numbers can be divided exactly by the multiple. Have students test their conjectures by having them perform the division calculation.

**LEARNING CONNECTION 3**

**Divisibility Challenge**

**MATERIALS**
- sheets of paper (1 per pair of students)

This activity is best completed after students have had an opportunity to investigate the divisibility of numbers (e.g., after completing Learning Connection 2).

Arrange students in pairs. Challenge them to work together to determine:
- a two-digit number that is divisible by 2;
- a three-digit number that is divisible by 2;
- a four-digit number that is divisible by 2;
- a five-digit number that is divisible by 2.

Next, have students find:
- a two-digit number that is divisible by 3;
- a three-digit number that is divisible by 3;
- a four-digit number that is divisible by 3;
- a five-digit number that is divisible by 3.

Continue the activity by having students determine two-, three-, four-, and five-digit numbers that are divisible by 4, by 5, and by 6.

**LEARNING CONNECTION 4**

**Striving for Small Remainders**

**MATERIALS**
- six-sided number cubes (1 number cube per pair of students)
- paper (a few sheets per pair of students)
Organize students into pairs. Explain the activity:

- In this game, the player with fewer points will be the winner.
- Together, partners pick any three numbers between 50 and 100, and record them on a piece of paper.
- The first player rolls the number cube to determine a divisor. If the player rolls a 1, he or she rolls the number cube again.
- The player selects one of the numbers recorded on the piece of paper and divides it by the divisor shown on the number cube. Students may use any calculation method they want (e.g., using partial quotients, using an algorithm).
- The remainder determines the number of points that the player earns.

Example: A player rolls a 6 on the number cube and chooses 87 from the recorded numbers. After dividing 87 by 6 and calculating the answer of 14 R3, the player receives 3 points. If there is no remainder, the player receives no points.
- The second player takes a turn to determine the number of points he or she receives.
- The player with fewer points after five rounds wins the game.

After students have played the game, discuss the strategies they used to obtain the smallest possible remainders. Students might explain that they tried to select numbers from the piece of paper that are divisible by the divisor shown on the number cube (e.g., selecting an even number after rolling a 2; selecting a number ending in 0 or 5 after rolling a 5).

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on division concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Multiplication and Division (4 to 6)”, and then click the number to the right of it.
Sharing Pennies

Dear Parent/Guardian:

We have been learning about different ways to solve division problems. Ask your child to solve the following problem.

Four children collected 627 pennies. They want to share the pennies equally.

• How many pennies will each child get?
• How many pennies will be left over?
• How many more pennies will they need to collect so that they all have the same number and no pennies are left over?

Have your child explain how he or she solved the problem. Discuss other ways to solve the problem. Thank you for doing this activity with your child.
### Hundreds Chart

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Grade 6 Learning Activity  
Gearing Up for a Biking Trip  

OVERVIEW  
In this learning activity, students are given the distance of a cycling trip (1550 km) and the average distance two cyclists can travel per day (95 km per day). They are asked to determine the number of days the trip will take. The emphasis in this learning activity is on interpreting the problem situation, applying meaningful procedures rather than simply using an algorithm, and making sense of the solution.

BIG IDEAS  
This learning activity focuses on the following big ideas:  
Operational sense: Students solve a division problem by using strategies that make sense to them. They discuss and analyse the various strategies used, in order to judge their efficiency and accuracy.  
Relationships: An understanding of number relationships helps students solve the problem in this learning activity. For example, students need to think about how 1550 km can be broken down into 95 km parts, to determine the number of travel days.  
Proportional reasoning: Students’ work that involves rate (kilometres per day) contributes to their understanding of proportional reasoning.

CURRICULUM EXPECTATIONS  
This learning activity addresses the following specific expectations.  
Students will:  
• solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms);  
• represent relationships using unit rates.  
These specific expectations contribute to the development of the following overall expectations.  
Students will:  
• solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;  
• demonstrate an understanding of relationships involving percent, ratio, and unit rate.
ABOUT THE LEARNING ACTIVITY

MATERIALS
• sheets of paper (2 per group of 2 or 3 students)
• overhead transparency of Div6.BLM1: Gearing Up for a Biking Trip
• overhead projector
• sheets of chart paper or large sheets of newsprint (1 per group of 2 or 3 students)
• markers (a few per group of 2 or 3 students)
• Div6.BLM2: Detour to Edmonton (1 per student)
• Div6.BLM3: Finding Travel Times (1 per student)

MATH LANGUAGE
• divide • quotient
• division • remainder
• divisor • algorithm
• dividend

INSTRUCTIONAL SEQUENCING
This learning activity serves as an introduction to strategies for solving problems that involve the division of four-digit whole numbers by two-digit whole numbers. It should be used before students learn the division algorithms for four-digit whole numbers by two-digit whole numbers, although students might use algorithms that they have learned in previous grades.

ABOUT THE MATH
As students progress through the junior grades, they are expected to perform division computations with increasingly larger numbers. Traditionally, the approach to teaching computations was the same at each grade – teachers reviewed the procedures for the standard division algorithm and then had students practise the algorithm using number sizes that were consistent with grade-level expectations. Although some students mastered the steps in performing the standard algorithm, fewer were successful in doing so as the number size and complexity of the computations increased. Even fewer students really understood the meaning behind the steps in the algorithm; they simply followed a memorized procedure.

The traditional approach to teaching division computations reinforces a belief in students that the standard algorithm is the only correct way to solve division problems. Students, especially those who struggle with the algorithm, focus on performing each step of the algorithm correctly, rather than on understanding the problem and the meaning of the solution.

A lack of understanding is often apparent when students attempt to explain the meaning of a remainder in a division problem. In one study, 70 percent of the 45 000 Grade 8 students correctly performed the long division for the following problem.

"An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?"
However, some wrote that "31 remainder 12" buses were needed, or just 31 – ignoring the remainder. Only 23 percent of the total group gave the correct answer of 32 buses (Schoenfeld, 1987).

In the following learning activity, students are given the distance of a cycling trip (1550 km) and the average distance the cyclists can travel per day (95 km per day). They are asked to determine the number of days the trip will take. Working collaboratively, students have the opportunity to share their understanding of the problem, discuss possible approaches, and help one another arrive at a solution by using strategies that make sense to them.

The answer to the division computation is 16 with a remainder of 30. Because students solve the problem in ways that are meaningful to them, they are more likely to understand the significance of the "16" and the "30" than if they merely calculated an answer using the standard algorithm. Understanding that the remainder represents a quantity that is a part of the solution allows students to interpret, account for, and represent the remainder in an appropriate way. For example, students need to realize that the "30" represents the remaining kilometres that are not travelled if the cyclists bike 95 km per day for 16 days. (The cyclists would be 30 km away from their destination at the end of the 16th day.) To deal with this leftover amount, students might have the cyclists travel more than 95 km per day, or they might add another day of travel.

**GETTING STARTED**

Ask students: “How long do you think it would take you to bike 100 km?” Have a few students estimate the time it might take, and ask them to explain how they made their estimates.

Continue the discussion by asking: “What information would help you answer the question more accurately?” Students might suggest, for example, that it would be helpful to know an actual 100 km distance or an average biking speed (e.g., kilometres per hour). Explain to students that you will provide some information to help them refine their estimates. Display the following statements on the board or on chart paper.

- The distance from the school to the [local site] is 5 km.
- It takes about an hour for a typical recreational cyclist to bike 15 km to 20 km.

Divide the class into groups of two or three students. Instruct students to work in their groups to determine the length of time it would take to bike 100 km. Invite them to use information (from the displayed statements or based on their own knowledge) to determine a solution. Provide each group of students with a sheet of paper on which they can record their work. Ask them to record their solution and to be prepared to share it with the class.

As students work on the problem, examine the various strategies they are using. For example, students might:

- refer to a familiar 100 km distance (e.g., the distance between two nearby towns) and estimate the time it would take to bike the distance;
- estimate the time it takes to bike 5 km and multiply this time by 20;
- consider the time it takes a recreational cyclist to bike 20 km and multiply this time by 5.
When students have finished recording their solutions, ask different groups to present their strategies and solutions to the class. Attempt to include groups who used a variety of strategies. Discuss the variety of approaches by asking questions such as the following:

- "Which strategies are similar? How are they alike?"
- "Why do the solutions differ? Is it possible to have an exact time for the solution?"
- "Which solutions seem reasonable? Why do you think they are reasonable?"
- "Which strategy, do you think, provides the most accurate solution? Why?"
- "What variables or factors might affect the time it takes to bike 100 km?"

For the last question, students might respond that factors such as the terrain, the kind of bike and condition of the bike, the physical condition of the rider, and the weather will influence the amount of time it would take to bike 100 km.

**WORKING ON IT**

Tell students that they are going to solve a problem encountered by Ben and Jen, two cyclists who are planning a biking trip from Winnipeg to Lake Louise. Have students locate these places on a map.

Display an overhead transparency of Div6.BLM1: Gearing Up for a Biking Trip, and discuss the problem:

"The distance from Winnipeg to Lake Louise, travelling west on the Trans-Canada Highway through Calgary, is 1550 km. From past experiences, Ben and Jen know that they can bike an average of 95 km/day. If they cycle at this speed, how many days will it take them to complete the trip?"

Arrange students in groups of two or three. Explain that each group will work to solve the problem in a way that makes sense to all its members. Provide each group with a sheet paper on which they can record their work.

Allow sufficient time for students to solve the problem and to record their strategies and solutions. Observe the strategies that students use, and provide guidance when necessary. If some students use a division algorithm, remind them that they need to be prepared to explain how and why the algorithm works. If they are unable to do so, suggest that they find a method that they are able to explain.

Move from group to group, and ask questions that encourage students to reflect on and articulate their reasoning:

- "What strategy are you using to solve this problem? Why did you choose this strategy?"
- "Is there a remainder? What does the remainder mean? How can you use the remainder in your solution?"
- "Is your answer reasonable? How do you know?"
- "How can you show your work so that others will understand what you are thinking?"
STRATEGIES STUDENTS MIGHT USE

USING REPEATED SUBTRACTION
Students might begin with 1550 and repeatedly subtract 95 until they reach a remainder of 30. They then count the number of times 95 was subtracted.

\[
\begin{array}{c}
1550 \\
- 95 \\
1455 \\
- 95 \\
1360 \\
- 95 \\
1265 \\
- 95 \\
\vdots
\end{array}
\]

and so on

USING DOUBLING
Students might double 95 to calculate the distance travelled in 2 days, and then continue to double the distance and the number of days until they reach a distance close to 1550.

\[
\begin{array}{c}
95 + 95 = 190 \text{ (2 days)} \\
190 + 190 = 380 \text{ (4 days)} \\
380 + 380 = 760 \text{ (8 days)} \\
760 + 760 = 1520 \text{ (16 days)} \\
1550 - 1520 = 30 \text{ (30 kilometres more to travel)}
\end{array}
\]

USING "CHUNKING"
Students might subtract “chunks” (multiples of 95) from 1550.

\[
\begin{array}{c}
1550 \\
- 950 \\
600 \\
- 190 \\
410 \\
- 190 \\
220 \\
- 190 \\
30
\end{array}
\]

(10 days)  
(2 days)  
(2 days)  
(2 days)
USING AN ALGORITHM
Students might use an algorithm to divide 1550 by 95.

<table>
<thead>
<tr>
<th>95</th>
<th>1550</th>
</tr>
</thead>
<tbody>
<tr>
<td>950</td>
<td>10</td>
</tr>
<tr>
<td>600</td>
<td>2</td>
</tr>
<tr>
<td>190</td>
<td>2</td>
</tr>
<tr>
<td>220</td>
<td>2</td>
</tr>
<tr>
<td>190</td>
<td>2</td>
</tr>
<tr>
<td>30</td>
<td>16</td>
</tr>
</tbody>
</table>

Note: If students attempt to use an algorithm that they have learned in previous grades, encourage them to think about the meaning of each procedural step.

When students have solved the problem, provide each group with markers and a sheet of chart paper or large sheet of newsprint. Ask students to record their strategies and solutions on the paper and to clearly demonstrate how they solved the problem.

Make a note of the various strategies used by students, and consider which groups might present their strategies during Reflecting and Connecting. Aim to include a variety of strategies (e.g., using repeated subtraction, using doubling, using “chunking”, using an algorithm).

REFLECTING AND CONNECTING
After students have finished solving the problem and recording their solutions, bring the class together to share their work. Ask a few groups of students to explain their strategies and solutions to the class. Pose guiding questions to help students explain their procedures:

• “What strategy did you use to solve the problem? Why did you use this strategy?”
• “How did you know that you were on the right track?”
• “Did you alter your strategy as you worked on the problem?”
• “What is your solution to the problem?”
• “Is the solution to the problem reasonable? How do you know?”
• “What did you do with the remainder?”

It is important that students have an opportunity to examine and discuss various strategies and evaluate their efficiency in terms of ease of use and effectiveness, in order to provide an accurate and meaningful solution. The purpose of this evaluation is not to have the class make definitive conclusions about which strategies are best, but to allow students, individually, to make decisions about which strategies make sense to them.
Encourage students to consider the effectiveness and efficiency of each strategy by asking the following questions after each presentation:

- “Was it easy to find a solution using your strategy?”
- “What are the advantages of this method? What are the disadvantages?”
- “How would you change your strategy if you solved the problem again?”

Conduct a think-pair-share activity. Provide 30 seconds for students to think about the different strategies they observed and to choose the strategy that they think worked best to solve the problem. Next, have them share their thoughts with a partner.

Ask a few students to share their thoughts about effective strategies with the class. Pose the following questions:

- “In your opinion, which strategy worked well?”
- “Why is the strategy effective in solving this kind of problem?”
- “How would you explain this strategy to someone who has never used it?”

ADAPTATIONS/EXTENSIONS

Simplify the problem for students who experience difficulties because of the size of numbers in the problem (e.g., “How many days will it take Ben and Jen to complete a trip of 260 km if they travel 65 km each day?”). It may be necessary to demonstrate a simple strategy, such as repeated subtraction, or to pair students with classmates who can explain a simple problem-solving method.

For students who require a challenge, ask them to solve the following problems:

- If Ben and Jen were to cycle 45 km in 3 1/2 hours, about how many kilometres would they cycle in 8 hours?
- If a 4-day cycle trip costs approximately $635, about how much would Ben and Jen spend on their trip from Winnipeg to Lake Louise?

ASSESSMENT

Observe students as they solve the problem to assess how well they:

- understand the problem;
- use an appropriate problem-solving strategy;
- judge the efficiency and accuracy of their strategy;
- solve the problem;
- explain the meaning of the remainder within the context of the problem and their solutions;
- explain their strategies and solutions clearly and concisely, using mathematical language;
- determine whether the solution is reasonable.

Provide an additional assessment opportunity by having students solve an additional problem.

Provide students with copies of Div4.BLM2: Detour to Edmonton, and discuss the problem.

“If Ben and Jen take the Yellowhead Highway from Winnipeg to Edmonton and then travel south to Lake Louise, the total distance is 1910 km. If Ben and Jen travel at a more leisurely pace of 85 km a day, how many days will it take them to complete the trip?”

Number Sense and Numeration, Grades 4 to 6 – Volume 4
Encourage students to think about the various strategies that the class used to solve the previous problem, and to apply one that would work well to solve this problem. Remind students to show their strategy and solution clearly so that others will know what they are thinking.

Observe students’ completed work and assess how well they apply an appropriate strategy, solve the problem, and explain their strategy and solution.

HOME CONNECTION

Send home Div6BLM3: Finding Travel Times. In this Home Connection activity, students solve a problem in which they determine the time it takes to travel by car between two cities and discuss their strategies with their parents.

LEARNING CONNECTION 1

Exploring a Flexible Division Algorithm

Learning the standard North American division algorithm can be difficult for students if they do not know basic multiplication facts, or if they are unsure of the steps involved in the algorithm. Exploring non-traditional algorithms provides students with an alternative to the standard North American algorithm and can help them understand the processes of division.

In the flexible algorithm explained below, students use known multiplication facts to determine parts that can be subtracted from the dividend. Students repeatedly subtract parts from the dividend until no multiples of the divisor are left. Students keep track of the pieces as they are subtracted, to the right of the algorithm.

Record the following on the board, and explain that the structure will allow students to calculate $1450 \div 43$.

Ask: “Is there at least one group of 43 in 1450? Are there at least 2 groups? At least 10 groups?”

When students agree that there are at least 10 groups of 43 (since $10 \times 43 = 430$, and 430 is less than 1450), complete the next step in the algorithm.
Explain that 1020 remains, and ask: "How many groups of 43 could we take from 1020?" Students might explain that another 10 groups of 43 could be taken from 1020. Record the next step in the algorithm.

\[
\begin{array}{c|c}
43 & 1450 \\
    & 430 \quad 10 \\
    & 1020 \\
    & 430 \quad 10 \\
    & 590 \\
\end{array}
\]

Continue to have students subtract multiples of 43 until no more multiples of 43 remain.

\[
\begin{array}{c|c}
43 & 1450 \\
    & 430 \quad 10 \\
    & 1020 \\
    & 430 \quad 10 \\
    & 590 \\
    & 430 \quad 10 \\
    & 160 \\
    & 86 \quad 2 \\
    & 74 \\
    & 43 \quad 1 \\
    & 31 \\
\end{array}
\]

After the algorithm has been completed, ask:
• "How many groups of 43 are there in 1450?"
• "What is the remainder?"
• "Why is there a remainder?"

Provide other opportunities for students to use the flexible algorithm.

**LEARNING CONNECTION 2**
**Making Sense of Remainders**

**MATERIALS**
• sheets of paper (1 per group of 2 or 3 students)

Solutions to division problems often involve remainders. The way in which remainders are dealt with depends on the context in the problem situation. For example, remainders can:
• be discarded;
• be partitioned into fractional pieces and distributed equally;
• remain a quantity;
• force the answer to the next highest whole number.
In other situations, the quotient can be rounded to the nearest whole number for an approximate answer. (See p. 17 for examples of different ways of dealing with remainders.)

This learning connection provides an opportunity for students to think about the meaning of a remainder within the context of a problem.

Organize students into groups of two or three. Ask each group to compose a word problem that involves $162 \div 12$, and to record it on a sheet of paper. Next, have groups exchange papers. Ask students to solve the problem in a way that makes sense to all group members. (See pp. 18–19 for possible strategies.)

Observe students as they solve the problem, and ask:

• "What strategy are you using to solve the problem?"
• "How do you know that this strategy is working?"
• "Is there some way to modify your strategy so that it will work better?"
• "Is there a remainder? How will you deal with the remainder so that it makes sense in your solution?"

Have groups present their strategies and solutions to the class. Discuss the meaning of the quotient and remainder within the context of each problem. Compare the different ways in which the remainder is dealt with in different situations.

**LEARNING CONNECTION 3**

**Asking Questions**

**MATERIALS**

• **Div6.BLM4: Asking Questions** (1 per pair of students)

Provide each pair of students with a copy of **Div6.BLM4: Asking Questions**. Explain that the page provides the answers to four questions, and that students are to determine what the questions might be, based on the information given at the top of the page. Have students work with their partner to discuss possible questions and to record them on the page.

Have pairs of students share their questions with the class.

Some possibilities are:

• What is the question if the answer is $16.50$? (How much did Joe earn per hour?)
• What is the question if the answer is 48? (How many hours did Joe work?)
• What is the question if the answer is $66$? (How much did Joe earn each day?)
• What is the question if the answer is $132$? (How much did Joe earn in 2 days?)
LEARNING CONNECTION 4
Base Ten Towers

MATERIALS
- base ten blocks, including hundreds flats, tens rods, and ones cubes (a collection for each group of 2 or 3 students)
- sheets of paper (1 per group of 2 or 3 students)
- pencils
- metre sticks (1 per group of 2 or 3 students)

Divide students into groups of two or three. Invite each group to build a tower using base ten blocks. Allow five minutes for students to build their towers. Challenge groups to calculate the cost of their towers if each hundreds flat is worth $100, each tens rod is worth $10, and each ones cube is worth $1.

Provide each group with a metre stick, and ask students to calculate the cost of each centimetre of the structure’s height.

Invite groups to explain the strategies they used throughout the activity.

eWORKSHOP CONNECTION
Visit www.eworkshop.on.ca for other instructional activities that focus on division concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Multiplication and Division (4 to 6)”, and then click the number to the right of it.
Gearing Up for a Biking Trip

The distance from Winnipeg to Lake Louise, travelling west on the Trans-Canada Highway through Calgary, is 1550 km. From past experiences, Ben and Jen know that they can bike an average of 95 km/day. If they cycled at this speed, how many days will it take them to complete the trip?
Detour to Edmonton

If Ben and Jen take the Yellowhead Highway from Winnipeg to Edmonton and then travel south to Lake Louise, the total distance is 1910 km. If Ben and Jen travel at a more leisurely pace of 85 km a day, how many days will it take them to complete the trip?
Finding Travel Times

Dear Parent/Guardian:

We have been learning about different ways to solve division problems. In math class, we solved a problem that involved finding the number of days it would take to bike 1550 km (kilometres) at a speed of 95 km per day. Students were encouraged to use methods that made sense to them, rather than follow a procedure that they might not understand. We then examined several ways to solve this problem and discussed the advantages and disadvantages of each method.

Have your child solve the following problem in a way that makes sense to him or her.

The distance from Barrie to Thunder Bay is 1275 km. How long would it take to travel this distance by car if you travel at an average speed of 85 km per hour?

Ask your child to explain how he or she solved the problem. You might also demonstrate how you would solve the problem.

As an extension activity, have your child find the distance between two provincial capitals, and have him or her determine the approximate time it would take to travel by car between the two cities.

Thank you for doing this activity with your child.
Asking Questions

Joe earned $792 for 12 days of work. Each day, he worked 4 hours.

• What is the question if the answer is $16.50?

• What is the question if the answer is 48?

• What is the question if the answer is $66?

• What is the question if the answer is $132?
Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.
INTRODUCTION

*Number Sense and Numeration, Grades 4 to 6* is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories behind good instruction that are elaborated in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The guide comprises the following volumes:
- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

The present volume – Volume 5: Fractions – provides:
- a discussion of mathematical models and instructional strategies that support student understanding of fractions;
- sample learning activities dealing with fractions for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume also contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by “eLearning modules” that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pp. 37, 49, and 67).
Relating Mathematics Topics to the Big Ideas

The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the “big ideas” of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005, p. 4)

In planning mathematics instruction, teachers generally develop learning activities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- operational sense
- proportional reasoning
- relationships

Each big idea is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies

- connecting
- representing
- communicating

Number Sense and Numeration, Grades 4 to 6 – Volume 5
The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

**Problem Solving:** Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

**Reasoning and Proving:** The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

**Reflecting:** Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

**Selecting Tools and Computational Strategies:** Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.
Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students’ own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The following table outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.
### Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
</table>
| Intellectual        | Generally, students in the junior grades:  
• prefer active learning experiences that allow them to interact with their peers;  
• are curious about the world around them;  
• are at a concrete operational stage of development, and are often not ready to think abstractly;  
• enjoy and understand the subtleties of humour. | The mathematics program should provide:  
• learning experiences that allow students to actively explore and construct mathematical ideas;  
• learning situations that involve the use of concrete materials;  
• opportunities for students to see that mathematics is practical and important in their daily lives;  
• enjoyable activities that stimulate curiosity and interest;  
• tasks that challenge students to reason and think deeply about mathematical ideas. |
| Physical            | Generally, students in the junior grades:  
• experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys);  
• are concerned about body image;  
• are active and energetic;  
• display wide variations in physical development and maturity. | The mathematics program should provide:  
• opportunities for physical movement and hands-on learning;  
• a classroom that is safe and physically appealing. |
| Psychological       | Generally, students in the junior grades:  
• are less reliant on praise but still respond well to positive feedback;  
• accept greater responsibility for their actions and work;  
• are influenced by their peer groups. | The mathematics program should provide:  
• ongoing feedback on students’ learning and progress;  
• an environment in which students can take risks without fear of ridicule;  
• opportunities for students to accept responsibility for their work;  
• a classroom climate that supports diversity and encourages all members to work cooperatively. |
| Social              | Generally, students in the junior grades:  
• are less egocentric, yet require individual attention;  
• can be volatile and changeable in regard to friendship, yet want to be part of a social group;  
• can be talkative;  
• are more tentative and unsure of themselves;  
• mature socially at different rates. | The mathematics program should provide:  
• opportunities to work with others in a variety of groupings (pairs, small groups, large group);  
• opportunities to discuss mathematical ideas;  
• clear expectations of what is acceptable social behaviour;  
• learning activities that involve all students regardless of ability. |

(continued)
Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moral and ethical</td>
<td>Generally, students in the junior grades:</td>
<td>The mathematics program should provide:</td>
</tr>
<tr>
<td>development</td>
<td>• develop a strong sense of justice and fairness;</td>
<td>• learning experiences that provide equitable opportunities for participation by all students;</td>
</tr>
<tr>
<td></td>
<td>• experiment with challenging the norm and ask “why” questions;</td>
<td>• an environment in which all ideas are valued;</td>
</tr>
<tr>
<td></td>
<td>• begin to consider others’ points of view.</td>
<td>• opportunities for students to share their own ideas and evaluate the ideas of others.</td>
</tr>
</tbody>
</table>

(Adapted, with permission, from Making Math Happen in the Junior Grades. Elementary Teachers’ Federation of Ontario, 2004.)
LEARNING ABOUT FRACTIONS
IN THE JUNIOR GRADES

Introduction
The development of fraction concepts allows students to extend their understanding of numbers beyond whole numbers, and enables them to comprehend and work with quantities that are less than one. Instruction in the junior grades should emphasize the meaning of fractions by having students represent fractional quantities in various contexts, using a variety of materials. Through these experiences, students learn to see fractions as useful and helpful numbers.

PRIOR LEARNING
In the primary grades, students learn to divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., half, third, fourth). Students use concrete materials and drawings to represent and compare fractions (e.g., use fraction pieces to show that three fourths is greater than one half). Generally, students model fractions as parts of a whole, where the parts representing a quantity are less than one.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES
As in the primary grades, the exploration of concepts through problem situations, the use of models, and an emphasis on oral language help students in the junior grades to develop their understanding of fractions.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to fractions, listed in the table on p. 12.
### Curriculum Expectations Related to Fractions, Grades 4, 5, and 6

<table>
<thead>
<tr>
<th>By the end of Grade 4, students will:</th>
<th>By the end of Grade 5, students will:</th>
<th>By the end of Grade 6, students will:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Overall Expectations</strong></td>
<td><strong>Overall Expectation</strong></td>
<td><strong>Overall Expectations</strong></td>
</tr>
<tr>
<td>• read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100;</td>
<td>• read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers.</td>
<td>• read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;</td>
</tr>
<tr>
<td>• demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts.</td>
<td></td>
<td>• demonstrate an understanding of relationships involving percent, ratio, and unit rate.</td>
</tr>
<tr>
<td><strong>Specific Expectations</strong></td>
<td><strong>Specific Expectations</strong></td>
<td><strong>Specific Expectations</strong></td>
</tr>
<tr>
<td>• represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered;</td>
<td>• represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools and using standard fractional notation;</td>
<td>• represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools and using standard fractional notation;</td>
</tr>
<tr>
<td></td>
<td>• compare and order fractions (i.e., halves, thirds, fourths, fifths, tenths) by considering the size and the number of fractional parts;</td>
<td>• represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation;</td>
</tr>
<tr>
<td></td>
<td>• compare fractions to the benchmarks of 0, 1/2, and 1;</td>
<td>• determine and explain, through investigation using concrete materials, drawings, and calculators, the relationship among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents.</td>
</tr>
<tr>
<td></td>
<td>• demonstrate and explain the relationship between equivalent fractions, using concrete materials and drawings;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• count forward by halves, thirds, fourths, and tenths to beyond one whole, using concrete materials and number lines;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• determine and explain, through investigation, the relationship between fractions (i.e., halves, fifths, tenths) and decimals to tenths, using a variety of tools and strategies.</td>
<td></td>
</tr>
</tbody>
</table>

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)
The following sections explain content knowledge related to fraction concepts in the junior grades, and provide instructional strategies that help students develop an understanding of fractions. Teachers can facilitate this understanding by helping students to:

- model fractions as parts of a whole;
- count fractional parts beyond one whole;
- relate fraction symbols to their meanings;
- relate fractions to division;
- establish part-whole relationships;
- relate fractions to the benchmarks of 0, 1/2, and 1;
- compare and order fractions;
- determine equivalent fractions.

**Modelling Fractions as Parts of a Whole**

Modelling fractions using concrete materials and drawings allows students to develop a sense of fractional quantity. It is important that students have opportunities to use area models, set models, and linear models, and to experience the usefulness of these models in solving problems.

**Area Models**

In an area model, one shape represents the whole. The whole is divided into fractional parts. Although the fractional parts are equal in area, they are not necessarily congruent (the same size and shape).

A variety of materials can serve as area models.

- Fraction Circles
- Pattern Blocks
- Fraction Rectangles
- Square Tiles
Set Models

In a set model, a collection of objects represents the whole amount. Subsets of the whole make up the fractional parts. Students can use set models to solve problems that involve partitioning a collection of objects into fractional parts.

A variety of materials can serve as set models.

Linear Models

In a linear model, a length is identified as the whole unit and is divided into fractional parts. Line-segment drawings and a variety of manipulatives can be used as linear models.

Modelling fractions using area, set, and linear models helps students develop their understanding of relationships between fractional parts and the whole. It is important for students to understand that:

- all the fractional parts that make up the whole are equal in size;
- the number of parts that make up the whole determine the name of the fractional parts (e.g., if five fractional parts make up the whole, each part is a “fifth”).

Number Sense and Numeration, Grades 4 to 6 – Volume 5
Teachers need to provide experiences in which students explore the usefulness of different models in problem-solving situations:

• Area models are useful for solving problems in which a whole object is divided into equal parts.

• Set models provide a representation of problem situations in which a collection of objects is divided into equal amounts.

• Length models provide a tool for comparing fractions, and for adding and subtracting fractions in later grades.

**Counting Fractional Parts Beyond One Whole**

Once students understand how fractional parts (e.g., thirds, fourths, fifths) are named, they can count these parts in much the same way as they would count other objects (e.g., “One fourth, two fourths, three fourths, four fourths, five fourths, . . .”).

Activities in which students count fractional parts help them develop an understanding of fractional quantities greater than one whole. Such activities give students experience in representing improper fractions concretely and allow them to observe the relationship between improper fractions and the whole (e.g., that five fourths is the same as one whole and one fourth).

**Relating Fraction Symbols to Their Meaning**

Teachers should introduce standard fractional notation after students have had many opportunities to identify and describe fractional parts orally. The significance of fraction symbols is more meaningful to students if they have developed an understanding of halves, thirds, fourths, and so on, through concrete experiences with area, set, and linear models.

The meaning of standard fractional notation can be connected to the idea that a fraction is part of a whole – the denominator represents the number of equal parts into which the whole is divided, and the numerator represents the number of parts being considered. Teachers should encourage students to read fraction symbols in a way that reflects their meaning (e.g., read 3/5 as “three fifths” rather than “three over five”).

Students should also learn to identify proper fractions, improper fractions, and mixed numbers in symbolic notations:

• In proper fractions, the fractional part is less than the whole; therefore, the numerator is less than the denominator (e.g., 2/3, 3/5).

**Learning About Fractions in the Junior Grades**
• In **improper fractions**, the combined fractional parts are greater than the whole; therefore, the numerator is greater than the denominator (e.g., 5/2, 8/5).

• In **mixed numbers**, both the number of wholes and the fractional parts are represented (e.g., 4 1/3, 2 2/10).

**Relating Fractions to Division**

Students should have opportunities to solve problems in which the resulting quotient is a fraction. Such problems often involve sharing a quantity equally, as illustrated below.

“Suppose 3 fruit bars were shared equally among 5 children. How much of a fruit bar did each child eat?”

To solve this problem, students might divide each of the 3 bars into 5 equal pieces. Each piece is 1/5 of a bar.
After distributing the fifths to the 5 children, students discover that each child receives 3/5 of a bar. An important learning from this investigation is that the number of objects shared among the number of sets (children, in this case) determines the fractional amount in each set (e.g., 4 bars shared among 7 children results in each child getting 4/7 of a bar; 2 bars shared among 3 students results in each child getting 2/3 of a bar). This type of investigation allows students to develop an understanding of fractions as division.

When modelling fractions as division, students need to connect fractional notation to what is happening in the problem. In the preceding example, the denominator (5) represents the number of children who are sharing the fruit bars, and the numerator (3) represents the number of objects (fruit bars) being shared.

Establishing Part-Whole Relationships

Fractions are meaningful to students only if they understand the relationship between the fractional parts and the whole. In the following diagram, the hexagon is the whole, the triangle is the part, and one sixth (1/6) is the fraction that represents the relationship between the part and whole.

By providing two of these three elements (whole, part, fraction) and having students determine the missing element, teachers can create activities that promote a deeper understanding of part-whole relationships. Using concrete materials and/or drawings, students can determine the unknown whole, part, or fraction. Examples of the three problem types are shown below.

**FIND THE WHOLE**

“If this rectangle represents 2/3 of the whole, what does the whole look like?”

To solve this problem, students might divide the rectangle into two parts, recognizing that each part is 1/3. To determine the whole (3/3), students would need to add another part.
FIND THE PART

“If 12 counters are the whole set, how many counters are 3/4 of the set?”

To solve this problem, students might divide the counters into four equal groups (fourths), then recognize that 3 counters represent 1/4 of the whole set, and then determine that 9 counters are 3/4 of the whole set.

FIND THE FRACTION

“If the blue Cuisenaire rod is the whole, what fraction of the whole is the light green rod?”

To solve this problem, students might find that 3 light green rods are the same length as the blue rods. A light green rod is 1/3 of the blue rod.

Relating Fractions to Benchmarks

A numerical benchmark refers to a number to which other numbers can be related. For example, 100 is a whole-number benchmark with which students can compare other numbers (e.g., 98 is a little less than 100; 52 is about one half of 100; 205 is a little more than 2 hundreds). As students explore fractional quantities that are less than 1, they learn to relate them to the benchmarks 0, 1/2, and 1. Using a variety of representations allows students to visualize the relationships of fractions to these benchmarks.

Using Fraction Circles (Area Model)
As students develop a sense of fractional quantities, they can use reasoning to determine whether fractions are close to 0, 1/2, or 1.

- In 1/8, there is only 1 of 8 fractional parts. The fraction is close to 0.
- One half of 8 is 4; therefore, 4/8 is equal to 1/2. 5/8 is close to (but greater than) 1/2.
- Eight eighths (8/8) represents one whole (1). 7/8 is close to (but less than) 1.

**Comparing and Ordering Fractions**

The ability to determine which of two fractions is greater and to order a set of fractions from least to greatest (or vice versa) is an important aspect of quantity and fractional number sense.

Students’ early experiences in comparing fractions involve the use of concrete materials (e.g., fraction circles, fraction strips) and drawings to visualize the difference in the quantities of two fractions. For example, as the diagram on p. 20 illustrates, students could use fraction circles to determine that 7/8 of a pizza is greater than 3/4 of a pizza.
As students use concrete materials to compare fractions, they develop an understanding of the relationship between the number of pieces that make the whole and the size of the pieces. Simply telling students that "the bigger the number on the bottom of a fraction, the smaller the pieces are" does little to help them understand this relationship. However, when students have opportunities to represent fractions using materials such as fraction circles and fraction strips, they can observe the relative size of fractional parts (e.g., eighths are smaller parts than fourths). An understanding about the size of fractional parts is critical for students as they develop reasoning strategies for comparing and ordering fractions.

Students can use several strategies to reason about the relative size of fractions.

- **Same-size parts**: The size of the parts (sixths) is the same for both fractions. Therefore, $\frac{4}{6} < \frac{5}{6}$.

- **Same number of parts but different-sized parts**: Fourths are larger parts than sixths. Therefore, $\frac{3}{4} > \frac{3}{6}$.

- **Nearness to one half**: $\frac{4}{8}$ is greater than one half ($\frac{1}{2}$). $\frac{1}{8}$ is less than one half ($\frac{1}{2}$). Therefore, $\frac{4}{8} > \frac{1}{8}$.

- **Nearness to one whole**: Eighths are smaller than fourths, so $\frac{7}{8}$ is closer to one whole than $\frac{3}{4}$ is. Therefore, $\frac{7}{8} > \frac{3}{4}$.
The strategies that students use to compare fractions (i.e., using concrete materials, using reasoning) can be applied to ordering three or more fractions. In a problem situation in which students need to order 3/5, 3/8, and 5/6, students might reason in the following way:

- Since eighths are smaller parts than fifths, 3/8 is less than 3/5.
- Since 5/6 is closer to 1 than 3/5 is, 5/6 is greater than 3/5.
- The fractions ordered from least to greatest are 3/8, 3/5, 5/6.

**Determining Equivalent Fractions**

Fractions are equivalent if they represent the same quantity. For example, in a bowl of eight fruits containing two oranges and six bananas, 2/8 or 1/4 of the fruits are oranges; 2/8 and 1/4 are equivalent fractions.

Students’ understanding of equivalent fractions should be developed in problem-solving situations rather than procedurally. Simply telling students to “multiply both the numerator and denominator by the same number to get an equivalent fraction” does little to further their understanding of fractions or equivalence.

Students can explore fraction equivalencies using area, set, and linear models.

### Finding Equivalent Fractions Using Area Models

Area models, such as fraction circles, fraction rectangles, and pattern blocks, can be used to represent equivalent fractions. Students can determine equivalent fractions by investigating which fractional pieces cover a certain portion of a whole. For example, as the following diagram illustrates, fraction pieces covering the same area of a circle demonstrate that 1/2, 2/4, 3/6, and 4/8 are equivalent fractions.

The following investigation involves using an area model to explore equivalent fractions.

- Cover this shape using one type of fraction piece at a time. Do not combine different types of pieces.
- Which types of fraction pieces cover the shape completely with no leftover pieces?
- Write a fraction for each of the ways you can cover the shape.
- What is true about these fractions?
Finding Equivalent Fractions Using Set Models

Students can use counters to determine equivalent fractions in situations that involve sets of objects. In the following diagram, counters show that \( \frac{1}{4} \) and \( \frac{3}{12} \) are equivalent fractions.

![Counter Diagram]

The following investigation involves using a set model to explore equivalent fractions.

"Arrange a set of 12 red counters and 4 yellow counters in equal-sized groups. All the counters within a group must be the same colour. How many different sizes of groups can you make? For each arrangement, record a fraction that represents the part that each colour is of the whole set."

Students might record the results of their investigation in a chart:

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>Fraction Red</th>
<th>Fraction Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3/12</td>
<td>1/4</td>
</tr>
<tr>
<td>8</td>
<td>6/16</td>
<td>2/8</td>
</tr>
<tr>
<td>16</td>
<td>12/16</td>
<td>4/16</td>
</tr>
</tbody>
</table>

The arrangement of counters in different-sized groups shows that \( \frac{3}{4} \), \( \frac{6}{8} \), and \( \frac{12}{16} \) are equivalent fractions, as are \( \frac{1}{4} \), \( \frac{2}{8} \), and \( \frac{4}{16} \).

Finding Equivalent Fractions Using Linear Models

Students can use fraction number lines to demonstrate equivalent fractions. All the following number lines show the same line segment from 0 to 1, but each is divided into different fractional segments. Equivalent fractions (indicated by the shaded bands) occupy the same position on the number line.
The following investigation involves using a set model to explore equivalent fractions.

"Use paper strips to find equivalent fractions. Create a poster that shows different sets of equivalent fractions."

A Summary of General Instructional Strategies

Students in the junior grades benefit from the following instructional strategies:

• partitioning objects and sets of objects into fractions, and discussing the relationship between fractional parts and the whole object or set;

• providing experiences with representations of fractions using area, set, and linear models;

• counting fraction pieces to beyond one whole using concrete materials and number lines (e.g., use fraction circles to count fourths: "One fourth, two fourths, three fourths, four fourths, five fourths, six fourths, ... ");

• connecting fractional parts to the symbols for numerators and denominators of proper and improper fractions;

• providing experiences of comparing and ordering fractions using concrete and pictorial representations of fractions;

• discussing reasoning strategies for comparing and ordering fractions;

• investigating the proximity of fractions to the benchmarks of 0, 1/2, and 1;

• determining equivalent fractions using concrete and pictorial models.

The Grades 4–6 Fractions module at www.eworkshop.on.ca provides additional information on developing fraction concepts with students. The module also contains a variety of learning activities and teaching resources.

eworkshop.on.ca
REFERENCES


Learning Activities for Fractions

Introduction

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to fractions. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or activity.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students’ understanding of mathematical concepts.
HOME CONNECTION: This section is addressed to parents or guardians and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.
Grade 4 Learning Activity
Every Vote Counts!

OVERVIEW
In this learning activity, students examine fractions that represent the results of a vote held by swim team members deciding whether or not to enter a swim meet. Students compare each fraction with the benchmarks of 0, 1/2, and 1 to determine whether at least half the team voted in favour of entering the meet.

BIG IDEAS
This learning activity focuses on the following big ideas:

Quantity: Students explore the “howmuchness” of different fractions by determining whether they are close to 0, 1/2, or 1.

Relationships: Students compare fractions with the benchmarks of 0, 1/2, and 1.

Representation: Students discuss the meaning of the numerator and the denominator in fraction representations.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations.
Students will:
• represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered;
• compare fractions to the benchmarks of 0, 1/2, and 1 (e.g., 1/8 is closer to 0 than to 1/2; 3/5 is more than 1/2).

These specific expectations contribute to the development of the following overall expectation.
Students will:
• read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100.
ABOUT THE LEARNING ACTIVITY

MATERIALS
- a chart with agree/disagree statements, written on the board or chart paper
- “Should We Enter the Swim Meet?” chart, written on the board or chart paper (for the Working on It part of the learning activity)
- a variety of manipulatives for representing fractions (e.g., fraction circles, counters, square tiles)
- half sheets of chart paper or large sheets of newsprint (1 per pair of students)
- markers (a few per pair of students)
- “Should We Enter the Swim Meet?” chart, written on the board or chart paper (for the Reflecting and Connecting part of the learning activity)
- Fra4.8LM1: Less Than, Equal to, or Greater Than 1/2 (1 per student)

MATH LANGUAGE
- fractional names (e.g., half, third, fourth)
- numerator
- denominator
- at least

INSTRUCTIONAL SEQUENCING
Before this learning activity, students should have had many experiences representing fractions using concrete materials (e.g., fraction circles, counters, square tiles) and drawings. This learning activity provides opportunities for students to compare fractions with the benchmarks of 0, 1/2, and 1 – a strategy that students can also apply when they compare fractions with other fractions (e.g., 3/8 is less than 1/2, and 4/5 is greater than 1/2; therefore, 3/8 is less than 4/5).

ABOUT THE MATH
Students develop a sense of fractional quantities by relating them to the benchmarks of 0, 1/2, and 1 (e.g., 1/8 is close to 0; 5/8 is close to 1/2; 7/8 is close to 1). Initially, students use concrete materials and drawings to determine the proximity of fractions to 0, 1/2, and 1. For example, they might use fraction circles as illustrated in the following diagrams.

- \( \frac{1}{8} \) of the fraction circle is covered. That is close to 0.
- \( \frac{1}{2} \) of the fraction circle is covered. That is close to 2.
- \( \frac{7}{8} \) of the fraction circle is covered. That is close to 1.
As students develop a stronger sense of fractional quantities, they can use reasoning strategies, such as the following, to determine whether fractions are close to 0, 1/2, or 1.

- In 1/8, there is only 1 of 8 fractional parts. The fraction is close to 0.
- One half of 8 is 4; therefore, 4/8 is equal to 1/2. 5/8 is close to (but greater than) 1/2.
- Eight eighths (8/8) represents one whole (1). 7/8 is close to (but less than) 1.

In this learning activity, students are asked to determine whether given fractions are less than or greater than 1/2. They are encouraged to use strategies that make sense to them – some students may use manipulatives or drawings to represent fractions, while others may use reasoning skills.

**GETTING STARTED**

Show the following chart, written on the board or chart paper, to the class.

<table>
<thead>
<tr>
<th>Agree</th>
<th>Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Taking a vote is the best way for a class to make a group decision.</td>
<td></td>
</tr>
<tr>
<td>2. In a class vote, the teacher should decide who may vote and who may not.</td>
<td></td>
</tr>
<tr>
<td>3. In a class vote, most students vote the same way as their friends.</td>
<td></td>
</tr>
</tbody>
</table>

Ask eight students to stand. Read the first statement in the chart, and ask the students who are standing to vote with their thumbs – using a thumbs-up gesture if they agree with the statement or using a thumbs-down signal if they disagree. Record the results on the chart (e.g., if 7 students agree and 1 student disagrees, write “7/8” in the Agree column and “1/8” in the Disagree column). Explain to the class that you used fractions to record the results of the vote. Ask the eight students to sit down.

Refer to the fraction in the “Agree” column, and ask:
- “What does the 8 mean?” (the number of students in the whole group)
- “What is the name for this part of the fraction?” (denominator)
- “What does the 7 mean?” (the number of students who agreed – part of the whole group)
- “What is the name for this part of the fraction?” (numerator)
- “How do we read this fraction?” (seven eighths)

Reinforce the meaning of denominator and numerator by asking similar questions about the fraction in the Disagree column.

Select six other students to stand. Ask these students to indicate using the thumbs-voting technique whether they agree or disagree with the second statement. Record the results on the chart using fractions expressed as sixths. Ask the six students to sit down.

Refer to the fractions recorded beside the second statement in the chart, and pose the following questions:
- “What do these two fractions mean?”
- “What is the denominator? Why is 6 the denominator?”
• “What are the numerators? Why?”
• “What fraction of the group agrees with the statement?”
• “Is this fraction close to none of the group, to half of the group, or to the whole group? How do you know?”
• “What fraction of the group disagrees with the statement?”
• “Is this fraction close to none of the group, to half of the group, or to the whole group? How do you know?”

Finally, ask 10 students to stand. Record the results of their voting for the third statement on the chart.

Use a think-pair-share strategy to have students reflect on and discuss the results of the vote for the third statement. Ask students to think about how the results of the vote could be interpreted using fraction language. Encourage them to think about whether the “agree” and “disagree” votes are close to none of the group, to half of the group, or to the whole group. Provide approximately 30 seconds for students to think individually, and then have them share their thoughts with a partner.

**WORKING ON IT**

Tell students the following:

“A swim meet is coming up. Teams may enter the meet if at least 1/2 of their team members agree to participate. Each team holds a vote to decide whether it will enter the meet.”

Display a partially completed chart with the names of the swim teams.

<table>
<thead>
<tr>
<th>Should We Enter the Swim Meet?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team</td>
</tr>
<tr>
<td>Dolphins</td>
</tr>
<tr>
<td>Marlins</td>
</tr>
<tr>
<td>Goldfish</td>
</tr>
</tbody>
</table>

Explain that the Dolphins team has 6 members, and that 4 members vote in favour of entering the meet. Ask: “What fraction of the Dolphins team agrees to enter the meet?” Record “4/6” in the Agree column beside Dolphins.

Complete the chart with students by explaining that 3 out of 7 Marlins team members voted to enter the meet (record “3/7” beside Marlins) and that 4 out of 8 Goldfish team members want to enter the meet (record “4/8” beside Goldfish).

Explain to students that their task is to determine which teams may enter the swim meet.
Organize students into pairs. Explain that students will work with a partner to solve the problem. Encourage them to use manipulatives (e.g., fraction circles, counters, square tiles) to help them think about the problem and a solution. Provide each pair of students with markers and a half sheet of chart paper or a large sheet of newsprint. Ask students to show how they solved the problem in a way that can be clearly understood by others.

Circulate around the room and observe students as they are working. Ask them questions such as the following:

• “What strategy are you using to solve the problem?”
• “How can you figure out whether 4/6 (3/7, 4/8) represents at least 1/2 of the team?”
• “How can you prove that your thinking is right?”

Students might use manipulatives and/or reasoning to determine whether the fractions are greater than 1/2.

**STRATEGIES STUDENTS MIGHT USE**

**USING MANIPULATIVES**

Students might use counters to represent the team members and separate the counters into two groups – one group to represent the “agree” members and the other group, the “disagree” members. For example, students could use 6 counters to represent the Dolphins team members. They might observe that 1/2 of 6 counters is 3 counters, so 4/6 is greater than 1/2.

For the Marlins team, students might separate 7 counters into a group of 3 and a group of 4. They might reason that 3 counters is 1/2 of 6 counters, therefore 3/7 is less than 1/2.

Using counters to model the outcome of the Goldfish team’s vote allows students to observe and represent equivalent fractions (4/8 is equal to 1/2).

**USING REASONING**

Students can use their knowledge of fractions to reason whether 4/6, 3/7, and 4/8 are greater than 1/2.

(continued)
For the Dolphins team, students might determine that 1/2 of 6 is 3 and conclude that 4/6 is greater than 1/2.

For the Marlins team, students might determine that 1/2 of 7 is 3 1/2 and decide, therefore, that 3/7 is less than 1/2. Or they might realize that 3 (the numerator in 3/7) is 1/2 of 6 and determine that 3/7 is less than 1/2.

For the Goldfish team, students might realize that 4 is 1/2 of 8 and recognize that 4/8 and 1/2 are equivalent fractions.

**REFLECTING AND CONNECTING**

Provide an opportunity for pairs of students to share their work and to explain their solutions to the whole class. Select pairs who used different strategies (e.g., using manipulatives, using reasoning), and allow students to observe various approaches to solving the problem. Make positive comments about students’ work, being careful not to infer that some approaches are better than others. Your goal is to have students determine for themselves which strategies are meaningful and efficient.

Post students’ work and ask questions such as:

- “What strategies are similar? How are they alike?”
- “Which strategy would you use if you solved another problem like this again?”
- “How would you change any of the strategies that were presented? Why?”
- “Which work clearly explains a solution? Why is the work clear and easy to understand?”

Provide another opportunity for students to relate fractions to the benchmarks of 0, 1/2, and 1 using reasoning strategies. Display the following chart, and explain that it shows the names of five other swim teams and the fraction of team members who voted in favour of entering the swim meet.

**Should We Enter the Swim Meet?**

<table>
<thead>
<tr>
<th>Team</th>
<th>Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clownfish</td>
<td>7/8</td>
</tr>
<tr>
<td>Barracudas</td>
<td>4/5</td>
</tr>
<tr>
<td>Guppies</td>
<td>2/5</td>
</tr>
<tr>
<td>Orcas</td>
<td>1/6</td>
</tr>
<tr>
<td>Sharks</td>
<td>4/10</td>
</tr>
</tbody>
</table>
Referring to each team in the chart, ask the following questions:

• “What fraction of the team voted in favour of entering the swim meet?”
• “Is the fraction closer to 0 (none of the team), 1/2, or 1 (the whole team)?”
• “How do you know that the fraction is closer to 0 (or 1/2 or 1)?”

Provide opportunities for several students to explain their thinking. Have them use manipulatives (e.g., fraction circles, counters, square tiles) to demonstrate their reasoning, thereby helping students who may have difficulty following oral explanations.

ADAPTATIONS/EXTENSIONS

Simplify the problem for students who might experience difficulties. For example: “8 out of 10 team members voted in favour of entering the swim meet. Did at least 1/2 of the team members vote in favour of entering the meet?” Encourage students to use manipulatives (e.g., fraction circles, counters, square tiles) to solve the problem.

Extend the problem for students who require a greater challenge:

“A coach agrees to enter teams in a swim meet if at least 1/2 of the members on each team vote in favour of doing so. Here are the numbers of team members who voted in favour of entering the meet:

- Mackerels: 12 out of 15
- Snappers: 9 out of 13
- Angelfish: 8 out of 14
- Trout: 11 out of 16

Which teams will enter the meet?”

ASSESSMENT

Have students, individually, solve the following problem. Ask students to record their solutions, reminding them to show their ideas in a way that can be clearly understood by others.

“Twelve members of a team are holding a vote to decide whether their team should enter a weekend competition. 7/12 of the team vote in favour of entering the competition. Is 7/12 closer to 0, 1/2, or 1? Explain your reasoning so that others will understand your thinking.”

Observe students’ work to assess how well they:

• determine that 7/12 is close to 1/2 (e.g., 6 is 1/2 of 12; therefore, 7/12 is close to 1/2);
• communicate a strategy and solution clearly;
• use appropriate drawings and/or explanations to demonstrate their thinking.

HOME CONNECTION

Send home Fra4.BLM1: Less Than, Equal to, or Greater Than 1/2. In this Home Connection activity, students and parents/guardians find examples of fractions at home and compare these fractions with 1/2. In class, encourage students to share their drawings and explain how their fraction examples compare with one half.
LEARNING CONNECTION 1
One Half as a Benchmark

MATERIALS
- a variety of fraction models, including area models (e.g., fraction circles, pattern blocks), set models (e.g., two-colour counters), and linear models (e.g., fraction strips, Cuisenaire rods).
See pp. 13–14 for other examples of area, set, and linear models.

Show students different representations of fractions, including area, set, and linear models. For each fraction, ask students to describe what the whole looks like. Next, ask students to determine whether each fraction is less than, equal to, or more than 1/2. Have students explain their reasoning.

LEARNING CONNECTION 2
Between 2/3 and 1

MATERIALS
- a variety of manipulatives for representing fractions (e.g., fraction circles, two-colour counters, Cuisenaire rods, square tiles)
- paper (1 per pair of students)

Have pairs of students find and record fractions that are between 2/3 and 1. Encourage students to use manipulatives (e.g., fraction circles, two-colour counters, Cuisenaire rods, square tiles) and drawings to help them.

Ask a few pairs to share their work with the class. Challenge students to prove that their fractions are between 2/3 and 1.

LEARNING CONNECTION 3
Making the Whole

MATERIALS
- Math Curse by Jon Scieszka
- manipulatives for representing fractions (e.g., fraction circles, fraction strips)

Read Math Curse by Jon Scieszka (New York: Viking Books, 1995), if available. In this book, the character sees everything in the world as a math problem. Towards the end of the book, the character is trapped in a room with a board that is covered with “a lifetime of problems”. The character breaks a stick of chalk in two and then puts the two halves of chalk together to make one whole. With a play on words, “whole” becomes “hole”, and the character escapes through a hole in the wall.
Ask students: "What fraction would I need to add to 1/4 to make 1 whole? How do you know?"
Encourage students to use drawings (e.g., circles or rectangles divided into parts) and manipulatives (e.g., fraction circles, fraction strips) to explain their thinking.

Provide other fractions (e.g., 2/3, 4/5, 1/6, 5/8), and ask students to determine the fraction that must be added to each to make one whole. Have students explain their thinking.

eWORKSHOP CONNECTION
Visit www.eworkshop.on.ca for other instructional activities that focus on fraction concepts. On the home page, click "Toolkit". In the "Numeracy" section, find "Fractions (4 to 6)", and then click the number to the right of it.
Dear Parent/Guardian:

We have been learning about ways to decide whether a fraction is close to 0, 1/2, or 1.

With your child, find examples of fractions in your home (e.g., the fraction of socks in a drawer that are black; the fraction of doors that have locks; the fraction that describes the amount of water in a bottle). Ask your child to compare these fractions with 1/2 (e.g. more than 1/2 of the socks in the drawer are black; exactly 1/2 of the doors have locks; the water bottle is less than 1/2 full).

Ask your child to make drawings to show how some of the fractions you found compare with 1/2.

In class, students will share their drawings and explain how their fraction examples compare with 1/2.

Thank you for doing this activity with your child.
Grade 5 Learning Activity
Investigating Fractions Using Tangrams

OVERVIEW
In this learning activity, students explore the fractional relationships between tangram pieces, and between each piece and the whole tangram square. Students also investigate equivalent fractions using different tangram pieces to represent 1/4, 1/2, and 3/4 of the whole tangram square.

BIG IDEAS
This learning activity focuses on the following big ideas:

Quantity: Students explore fractional quantities by identifying the fraction that each tangram piece represents of the whole tangram square, by comparing the sizes of different fractions, and by representing equivalent fractions.

Relationships: Students explore the relationships between equivalent fractions (e.g., multiplying the numerator and denominator of a fraction by the same number produces an equivalent fraction).

Representation: Students observe that a fractional amount can be represented by equivalent fractions (e.g., 1/4, 2/8, and 4/16 represent the same area of the whole tangram square).

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations.

Students will:
- represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, number lines) and using standard fractional notation;
- demonstrate and explain the concept of equivalent fractions, using concrete materials (e.g., use fraction strips to show that 3/4 is equal to 9/12).

These specific expectations contribute to the development of the following overall expectation.

Students will:
- read, represent, compare, and order whole numbers to 100,000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers.
ABOUT THE LEARNING ACTIVITY

MATERIALS
- overhead projector
- overhead transparency of tangram pieces
- sets of tangrams, either commercially produced or cut out of stiff paper using
  Fra5.BLM1: Tangram Puzzle as a template (1 set per student)
- Fra5.BLM2: Tangram Square (1 per pair of students)
- Fra5.BLM3: Tangram Fractions (1 copy)
- instructions for Working on It Activity 1 posted on the board or chart paper
- Fra5.BLM4: 1/4 Tangram Square (1 per pair of students)
- sheets of paper (several per pair of students)
- scissors (1 pair per pair of students)
- Fra5.BLM5: 1/2 Tangram Square (1 per pair of students)
- Fra5.BLM6: 3/4 Tangram Square (1 per pair of students)
- Fra5.BLM7: Finding Equivalent Fractions (1 per student)
- sheets of paper or math journals (1 per student)

MATH LANGUAGE
- fractional names (e.g., half, fourth, eighth, sixteenth)
- numerator
- denominator
- equal/equivalent

INSTRUCTIONAL SEQUENCING
Before this learning activity, students should have had experiences dividing whole objects into equal parts and identifying the parts using fractional language (e.g., “three fourths”) and standard fractional notation (e.g., 3/4). The following activity provides an opportunity for students to represent fractions concretely and to investigate equivalent fractions.

ABOUT THE MATH
In this learning activity, students explore the fractional relationship between each tangram piece and the whole tangram square (e.g., the large triangles each represent 1/4 of the tangram square; the medium-sized triangle represents 1/8 of the square; the small triangles each represent 1/16 of the square).

Students also find different ways to represent 1/4, 1/2, and 3/4 of the whole tangram square using different tangram pieces. For example, they discover that 1/4 of the whole tangram square can be covered using one of the large triangles (1/4 of the whole tangram square) or two medium-sized triangles (2/8 of the square) or four small triangles (4/16 of the square). This discovery leads to a discussion about equivalent fractions as different fractions that represent
the same quantity (same area). Students investigate the notion that multiplying the numerator
and the denominator of a fraction by the same number produces an equivalent fraction.

Note: In this activity, fractions represent the areas of the tangram pieces compared with a whole
tangram (e.g., the area of the large triangle represents 1/4 of the area of the whole tangram
square). Be aware that some students might think about the fractions in terms of the number
of tangram pieces compared with the whole (e.g., since there are 7 tangram pieces, each piece
is 1/7 of the whole tangram). Focus these students’ attention on the relative areas of the tangram
pieces. For example, you might have students compare the areas of the tangram pieces by asking
them to decide which piece has a bigger or smaller area. You might also have students super-
impose tangram pieces to find equal areas (e.g., two medium-sized triangles have the same
area as one large triangle).

GETTING STARTED

Tell the following story about the origin of the tangram puzzle.

“Many years ago, in China, there lived a man called Mr. Tan. Of all his possessions, he
most treasured an exquisite porcelain square tile. One day, he heard that the Emperor
of China was coming to his village. To show his great admiration for and loyalty to the
Emperor, Mr. Tan decided to offer his very precious tile to the Emperor as a gift. In great
excitement, he began to polish his tile so that it would shine. As he handled the tile in
different ways, to polish every surface, he dropped it. The porcelain tile broke into the
seven pieces of the tangram puzzle. Mr. Tan was so very unhappy. As he wiped away
his tears, he thought that if he could put the pieces back together, he would have the
square tile again. Mr. Tan thought it would be easy to do, but it took him a very long
time. While he was trying to form the square, he discovered lots of interesting two-
dimensional shapes.”

Using an overhead projector and an overhead transparency of tangram pieces (or pieces cut out
from Fra5.BLM1: Tangram Puzzle), show the following shapes of objects made using the seven
tangram pieces. Ask students to try to identify the objects.

![Tangram Shapes](image)

Provide each student with a set of tangram pieces. (If you don’t have enough sets of commercially
produced tangrams, distribute copies of Fra5.BLM1: Tangram Puzzle, and have students cut out
their own sets of tangram pieces.) Challenge students to construct a square using all seven tangram
pieces. Invite students who finish before others to find different ways to create the square.
Have students compare their square arrangement with a partner’s. Some students may observe that all the different arrangements are congruent and that some are rotated (turned) or reflected (flipped) versions of others.

Provide each pair of students with a copy of Fra5.BLM2: Tangram Square, and explain that the square outline on the page is the same size as the tangram square. (If the tangrams in your classroom form a different size of square from the one on Fra5.BLM2, you will need to resize the blackline master.) Ask: “How many large triangles would you need to cover the square?” Provide an opportunity for students to manipulate the tangram pieces (students may combine their sets of tangram pieces) and to determine that four large triangles would cover the square. Discuss how the large triangle represents 1/4 of the square. Post a copy of Fra5.BLM3: Tangram Fractions, and label “1/4” on each of both large triangles.

Next, challenge students to find the fractional relationship between each of the other tangram pieces and the whole tangram square.

After students have had sufficient time to conduct their investigations, ask the following questions:

• “How many medium-sized triangles do you need to cover the whole square?” (8)
• “What fraction of the whole square is the medium-sized triangle?” (one eighth)
• “How many small triangles would you need to cover the whole square?” (16)
• “What fraction of the whole square is one small triangle?” (one sixteenth)

On the posted copy of Fra5.BLM3: Tangram Fractions, label “1/8” on the medium-sized triangle and “1/16” on each of both small triangles.

Next, have pairs of students explain how they determined the fractional relationship between the square tangram piece and the whole tangram square. They might have discovered that the area of the square tangram piece is equal to the area of the two small triangles; therefore, its area is equal to that of the medium-sized triangle, which is 1/8 of the whole tangram square. Label “1/8” on the square tangram piece on the posted copy of Fra5.BLM3: Tangram Fractions.

Finally, discuss how the parallelogram has the same area as the medium-sized triangle and as the square tangram piece; therefore, it is 1/8 of the whole square. Label “1/8” on the parallelogram tangram piece on the posted copy of Fra5.BLM3: Tangram Fractions.

Ask students to examine the labelled copy of Fra5.BLM3: Tangram Fractions, and invite them to make observations. For example, students might observe that the square, the medium-sized triangle, and the parallelogram each represent 1/8 of the whole square even though they are different shapes. Students might also describe the relationships between shapes (e.g., that the parallelogram piece could be composed by combining two small triangles).
WORKING ON IT

ACTIVITY 1: FINDING EQUIVALENT REPRESENTATIONS OF 1/4

Provide each pair of students with a copy of Fra5BLM4: 1/4 Tangram Square. Explain to students that they are to find different ways to cover 1/4 of the whole square (the shaded area of the square). Ask students to use one kind of tangram piece (e.g., only small triangles) each time. Tell students that they may find arrangements that involve more tangram pieces than those in their set. For example, they may come up with an arrangement that involves four small triangles even though the tangram set contains only two small triangles.

Provide the following instructions (posted on the board or chart paper), and ask students to follow the process for each different arrangement:

• Cover 1/4 of the large square with identical tangram pieces.
• Arrange the identical tangram pieces on another sheet of paper, and trace around the arrangement.
• Cut out the arrangement (along only the outside lines to make one piece).
• Label each tangram piece within the cut-out with its fraction name.

Show students an example of a completed cut-out arrangement:

Gather the class after students have completed the activity. Have students show different cut-outs that cover 1/4 of the tangram square, and ask them to explain the fractions represented by the tangram pieces within each cut-out.

Note: If students propose arrangements other than those involving one large triangle, two medium-sized triangles, or four small triangles, ask them to flip or rotate their cut-outs to see if they are congruent to those already discussed.

Borrow the following cut-outs from students, and post them on the board or chart paper. Record the corresponding fraction below each cut-out.
Help students to recognize the equivalence of 1/4, 2/8, and 4/16 by asking the following questions:
- "What fraction of the large tangram square do these cut-outs represent? How do you know?"
- "How many eighths are equal to 1/4?"
- "How many sixteenths are equal to 1/4?"
- "How do you know that 1/4, 2/8, and 4/16 are equivalent fractions?" (They all represent the same quantity/area.)

**ACTIVITY 2: FINDING EQUIVALENT REPRESENTATIONS OF 1/2 AND 3/4**

Provide each pair of students with a copy of *Fra5.BLMS5: 1/2 Tangram Square* and *Fra5.BLMS6: 3/4 Tangram Square*. Explain that the activity is similar to the previous one but that this time students need to find different ways to cover 1/2 and 3/4 of the large square. Review the process of covering the shaded part of the whole tangram square with identical tangram pieces (one kind of tangram piece each time), tracing around the pieces on paper, cutting out the arrangement, and labelling the fractions.

After students have had sufficient time to find different representations for 1/2 and 3/4, ask each pair of students to join another pair. Have the groups discuss how they know their cut-outs represent 1/2 or 3/4.

Borrow the following cut-outs from students, and post them on the board or chart paper. Record the corresponding fraction below each cut-out.

Ask the following questions:
- "How do you know that all these cut-outs represent 1/2 of the large square?"
- "What are some equivalent fractions for 1/2?"

Next, post the following cut-outs for 3/4, and record the corresponding fraction below each cut-out.

![Fraction Cut-Outs](image-url)
Ask students to explain how they know that all the cut-outs represent $\frac{3}{4}$ of the whole tangram square. Discuss the equivalence of $\frac{3}{4}$, $\frac{6}{8}$, and $\frac{12}{16}$.

REFLECTING AND CONNECTING

Review the equivalent fractions for $\frac{1}{2}$ that the class found using tangram pieces, and record the following on the board or chart paper:

$$
\frac{1}{2} = \frac{2}{4} \quad \frac{1}{2} = \frac{4}{8} \quad \frac{1}{2} = \frac{8}{16}
$$

Ask students to describe any patterns they observe. Elicit the idea that when the numerator and denominator of $\frac{1}{2}$ are both multiplied by 2 or 4 or 8, this produces an equivalent fraction.

Next, record the equivalent fractions for $\frac{1}{4}$ and $\frac{3}{4}$.

$$
\frac{1}{4} = \frac{2}{8} \quad \frac{1}{4} = \frac{4}{16} \quad \frac{3}{4} = \frac{6}{8} \quad \frac{3}{4} = \frac{12}{16}
$$

Ask students whether the same idea of multiplying the numerator and the denominator by the same number applies to equivalent fractions for $\frac{1}{4}$ and $\frac{3}{4}$. Have students explain their thinking to a partner. Invite students to apply the rule to determine other equivalent fractions for $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.

Ask students to record their thoughts about the following question on a sheet of paper or in their math journals: “What do you know about equivalent fractions?”

ADAPTATIONS/EXTENSIONS

Some students may find it difficult to work with different tangram pieces at the same time. Give these students only one type of tangram piece at a time (e.g., only small triangles to represent sixteenths; only medium-sized triangles to represent eighths).

For students who require a greater challenge, ask them to use tangram pieces to find as many different examples of equivalent fractions as possible. For example, students might use 6 small triangles and 3 medium-sized triangles to show that $\frac{6}{16}$ and $\frac{3}{8}$ are equivalent fractions.

ASSESSMENT

As students are working with tangram pieces, ask the following questions to assess their understanding of fraction representations and relationships:

- "What fraction of the whole square is this tangram piece? How do you know?"
- "How could you show $\frac{1}{2}$ (or $\frac{1}{4}$ or $\frac{3}{4}$) of the whole square? How can you show this fraction in a different way?"
• “What fractions are equivalent to 1/4? To 1/2? To 3/4? How do you know that these fractions are equivalent?”

Examine students’ journal entries (completed in the Reflecting and Connecting portion of the learning activity) to assess how well they understand that equivalent fractions represent the same quantity using different-sized fractional parts.

HOME CONNECTION

Send home copies of Fractions: Finding Equivalent Fractions. In this Home Connection activity, students and their parents/guardians find examples of equivalent fractions in their home. In class, encourage students to share and explain their diagrams.

LEARNING CONNECTION 1
Equivalent Fractions With Set Models

MATERIALS
• two-colour (red, yellow) counters (14 per pair of students)

In this learning activity, students have explored equivalent fractions using an area model (tangrams). Students can also investigate equivalent fractions using a set model.

Provide pairs of students with two-colour counters. Have the students display 4 red counters and 6 yellow counters. Establish the idea that the entire set (the whole) is composed of all 10 counters and that 4/10 of the counters are red and 6/10 are yellow.

Next, have students rearrange the counters into equal-sized groups, where the counters within a group are the same colour, to find equivalent fractions.

Five groups showing 2 red and 4 yellow

Repeat the activity using:
• 3 red and 6 yellow;
• 6 red and 2 yellow;
• 4 red and 8 yellow.
LEARNING CONNECTION 2
Equivalent Fractions Game

MATERIALS
• copies of Fra5.BLM8: Fraction Strips (1 per student)
• scissors (1 pair per student)

Provide students with a copy of Fra5.BLM8: Fraction Strips, and instruct them to cut out the fraction strips and cut the pieces apart.

Arrange students in pairs, and explain the game:
• Players combine their fraction pieces in a common pile.
• Players take turns selecting fraction pieces until all pieces have been drawn.
• Each player arranges his or her fraction pieces to create pairs of equivalent fractions, for example:

![Fraction strips](image)

Each player identifies the equivalent fractions that he or she created (e.g., 1/4 = 2/8 and 1/2 = 4/8) and earns a point for each pair correctly identified. The player with more pairs of equivalent fractions wins the game.

LEARNING CONNECTION 3
Introducing Mixed Numbers

MATERIALS
• overhead transparency of pattern blocks, if available, or regular pattern blocks
• overhead projector

Mixed numbers represent quantities that comprise one or more wholes and fractional parts. The following activity allows students to see that quantities greater than 1 can be written as a fraction and as a mixed number.

Using overhead transparency pattern blocks on an overhead projector, display a yellow hexagon and explain that it represents one whole unit. Next, show a collection of 17 sixths (green triangles). Have students count the sixths orally as you point to each block (“One sixth, two sixths, three sixths, . . ., seventeen sixths”). Ask students how they might record the number of sixths using fractional notation (17/6).
Next, have a student regroup the sixths on the overhead projector into whole hexagon shapes. Again, ask students to explain how they might record the quantity shown on the overhead projector. For example, they might suggest the following forms:

- 2 wholes and 5/6
- 2 and 5/6
- 2 + 5/6

Emphasize that their suggestions are possible ways to record the amount and that 2 5/6 is the standard way to represent 2 wholes and 5/6.

Repeat the activity using other collections of fractional parts (e.g., red trapezoids for halves, blue rhombuses for thirds).

**LEARNING CONNECTION 4**  
**Improper Fractions and Mixed Numbers**

**MATERIALS**
- pattern blocks, including yellow hexagons, red trapezoids, blue rhombuses, and green triangles (several per pair of students)
- sheets of paper (1 per pair of students)

Provide each pair of students with several pattern blocks. Explain that the yellow hexagon pattern block represents one whole. Invite students to cover the surface of the yellow hexagons with red trapezoids, blue rhombuses, and green triangles. Establish that the red trapezoid is 1/2 of the yellow hexagon, the blue rhombus is 1/3 of the yellow hexagon, and the green triangle is 1/6 of the yellow hexagon.

Ask students to work with their partner to create an arrangement using one kind of pattern block (e.g., only green triangles) that represents a quantity greater than one whole. Each arrangement should consist of fewer than 10 pattern blocks.

Ask a few pairs to show their arrangement to the class and to identify the fraction represented (e.g., an arrangement with 7 green triangles represents 7/6).

Next, instruct students to substitute a yellow hexagon for the pattern blocks that equal the whole (e.g., in an arrangement of 7 green triangles, students would take out 6 triangles and substitute
a yellow hexagon, leaving a yellow hexagon and a green triangle). Have students rename their arrangement using a mixed number.

Have pairs of students create different pattern-block arrangements that are greater than 1. Ask them to record each arrangement by tracing around the pattern blocks on a sheet of paper and to label the arrangement using both an improper fraction and a mixed number, for example:

\[ \frac{5}{3} = 1 \frac{2}{3} \]

Observe students’ work to determine how well they are able to identify improper fractions and corresponding mixed numbers.

eWORKSHOP CONNECTION
Visit www.eworkshop.on.ca for other instructional activities that focus on fraction concepts. On the home, click “Toolkit”. In the “Numeracy” section, find “Fractions (4 to 6)”, and then click the number to the right of it.
Tangram Puzzle
Tangram Square
Tangram Fractions
1/4 Tangram Square
1/2 Tangram Square
3/4 Tangram Square
Finding Equivalent Fractions

Dear Parent/Guardian:

We are learning about equivalent fractions. Fractions are equivalent if they represent the same amount. For example, in the following diagram, 6 of the 12 fruits are apples. We could say that 6/12 or 1/2 of the fruits are apples; 6/12 and 1/2 are equivalent fractions.

Help your child find examples of equivalent fractions in your home. For instance, you might use a dozen eggs to show that 8/12 of the eggs is the same as 2/3 and 4/6 of the eggs.

Ask your child to draw a diagram that shows an example of equivalent fractions that you found in your home. Have your child label the equivalent fractions in the diagram, and ask him or her to explain why the fractions are equivalent.

In class, students will share their diagrams with their classmates.

Thank you for doing this activity with your child.
Grade 6 Learning Activity
Fraction Line-Up

OVERVIEW
In the following learning activity, students create proper fractions and improper fractions using numerals obtained by rolling a pair of number cubes. They compare and order the fractions by placing them on a number line.

BIG IDEAS
This learning activity focuses on the following big ideas:

Quantity: Students explore fractional quantity by ordering proper fractions and improper fractions on a number line.

Relationships: Students compare and order proper fractions and improper fractions, and relate them to whole numbers.

Representation: Modelling fractions concretely and pictorially and locating fractions on a number line help students to understand the quantity represented by fraction symbols.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations.

Students will:
• represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, drawings, number lines, calculators) and using standard fractional notation;
• determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents (e.g., use a 10 × 10 grid to show that 1/4 = 0.25 or 25%).

These specific expectations contribute to the development of the following overall expectation.

Students will:
• read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers.
ABOUT THE LEARNING ACTIVITY

MATERIALS
• sheets of paper (1 per group of 3 or 4 students)
• overhead transparency of Fra6.BLM1: 0-6 Number Line
• overhead transparency marker
• number cubes (2 per group of 3 or 4 students)
• number lines made with Fra6.BLM2: Number-Line Strip (1 per group of 3 or 4 students)
• small pieces of paper – approximately 4 cm x 6 cm (several per group of 3 or 4 students)
• fraction models (e.g., fraction circles)
• Fra6.BLM3: Fraction Number Line (1 per group of 3 or 4 students)
• large sheets of paper (2 per pair of students)
• Fra6.BLM4: Fraction Circles (a few copies)
• Fra6.BLM5: Ask Me About Fractions (1 per student)

MATH LANGUAGE
• fractional names (e.g., halves, thirds, fourths, …)
• fractional part
• proper fraction
• improper fraction
• compare
• order
• equal/equivalent

INSTRUCTIONAL SEQUENCING
Before Grade 6, students represent, compare, and order proper and improper fractions with like denominators. The following learning activity allows students to explore quantities associated with proper and improper fractions, and to order them on a number line.

ABOUT THE MATH
In Grade 6, students are expected to represent, compare, and order proper and improper fractions. They need to understand the meaning of these two kinds of fractions, not merely learn abstract rules (e.g., in proper fractions, the numerator is less than the denominator; in improper fractions, the numerator is greater than the denominator) that contribute little to students’ development of fraction sense. In particular, students should be able to:
• compare the fractional part of a proper or an improper fraction with the whole (e.g., in both 2/3 and 4/3, three thirds make a whole);
• recognize whether a fraction is less than, equal to, or greater than 1 (e.g., 2/3 is less than 1; 3/3 is equal to 1; 4/3 is greater than 1);
• express improper fractions as mixed numbers (e.g., 4/3 = 1 1/3).

In the following learning activity, students need to consider the size of proper and improper fractions in order to locate their approximate positions on a number line. The learning activity
Number Sense and Numeration, Grades 4 to 6 – Volume 5

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requires students to think about the proximity of fractions to the whole numbers (0 to 6) that are given on the number line. The task of locating fractions on a number line helps to develop students’ understanding of fractional quantities, relationships between fractions, and relationships between whole numbers and fractions.

GETTING STARTED

Relate the following anecdote to the class:

Owen and Teresa were playing a game with two regular number cubes. Each player took a turn rolling the number cubes, one at a time. The first number rolled was the number of tens, and the second number rolled was the number of ones. At the end of each round, the player who rolled the greatest number won a point. Teresa and Owen always hoped to roll 66 because they knew that was the greatest possible number they could obtain.

After a while, Owen proposed a new game. “What if we played the game and created fractions with the numbers we roll? The number on the first number cube would be the numerator, and the number on the second number cube would be the denominator.”

“Sounds like a good game,” said Teresa. “So, I wonder… what is the greatest possible fraction we could get?”

Owen responded, “Well, I know that three fourths is a big fraction, so maybe we would have to roll a 3 and a 4.”

Teresa wasn’t sure. “But what if we rolled a 6 both times? Wouldn’t six sixths be the greatest possible fraction?”

Pose the problem: “What is the greatest possible fraction you could create if you rolled two number cubes?”

Divide the class into groups of three or four. Provide each group with a sheet of paper and pencils. Ask students to discuss the problem and to record a solution. Encourage students to use diagrams in their solutions to help to explain their thinking.

After students have had an opportunity to discuss the problem and to record their solutions, gather the class and invite some groups to explain their thinking. As students present their solutions, assess how well they explain fractional quantities.

Some students may suggest that the best possible roll would be a 6 and a 1, giving a fraction of 6/1. Record “6/1” on the board, and ask:

• “What is the denominator in this fraction?” (1)
• “What does 1, as a denominator, represent?” (the whole)
• “What does 6/1 mean?” (6 wholes)
• “What does 6/1 look like using fraction circles?” (six whole circles)

On the board, draw 6 circles. Connect the diagram to the fraction symbol by reinforcing the ideas that the denominator represents 1 whole and that the numerator (6) tells the number of wholes.
Invite students to determine whether 6/1 is the greatest possible fraction that can be created using two number cubes. If students propose other possibilities, encourage them to use diagrams or manipulatives (e.g., fraction circles) to support their argument.

Display an overhead transparency of **Fra6.BLM1: 0–6 Number Line**, and emphasize the idea that the number line contains the whole numbers from 0 to 6. Explain to students that the number line can be used to order fractions. As an example, have students indicate the position of 1/2. If students suggest that 1/2 is located halfway along the line (i.e., where 3 is located), ask them to consider whether 1/2 and 3 are equivalent numbers. Emphasize that 1/2 is midway between 0 and 1, and record “1/2” on the number line using an overhead marker.

Next, have students consider where 3/4 (Owen’s fraction) would be located. Again, emphasize that the location of the fraction is determined by considering its proximity to the whole numbers on the line – not by considering the complete line as one whole. After students determine that 3/4 is located midway between 1/2 and 1, record “3/4” on the number line.

Continue by having students determine where 6/6 (Teresa’s fraction) and 6/1 would be located, and record these fractions on the number line.

**WORKING ON IT**

Explain the following activity:

- Students work in groups of three or four. Each group requires two number cubes and a number line made from **Fra6.BLM2: Number-Line Strip**.
- Each student in the group takes a turn rolling the number cubes, one at a time. The number on the first cube indicates the numerator of a fraction; the second cube indicates the denominator. For example, if the first cube shows 5 and the second cube shows 4, the fraction is 5/4. The student creates a fraction card by recording the fraction on a small piece of paper.
- After all students in the group have each created and recorded a fraction, they decide, collaboratively, where each fraction card should be placed on the number line. Students need to consider the proximity of each fraction to the whole numbers on the number line and to each other. Students should attempt to place the fraction cards, as accurately as possible, in their positions on the number line. For example, if students create fraction cards with 5/3, 2/3, 5/1, and 2/2, they place the cards in the following positions:

```
0 1 2 3 4 5 6
```

```
5/3 2/3 5/1 2/2
```

- Students may use fraction models (e.g., fraction circles, diagrams), their knowledge of equivalent fractions and whole numbers, and reasoning to help them place fraction cards in their correct positions on the number line.
STRAATEGIES STUDENTS MIGHT USE

USING FRACTION MODELS

Students might use manipulatives (e.g., fraction circles) and diagrams to represent fractions and to determine their proximity to whole numbers. For each fraction, students need to consider the size of the fractional part (the denominator) and the number of parts (the numerator). For example, \( \frac{5}{4} \) could be represented using a diagram or fraction circles. Students could use the representation to determine that \( \frac{5}{4} \) is \( \frac{1}{4} \) more than 1 and, accordingly, locate \( \frac{5}{4} \) on the number line.

![Fraction model example](image)

USING KNOWLEDGE OF FRACTION-WHOLE NUMBER EQUIVALENCIES

Students learn to recognize number patterns that help them identify equivalent numbers:
- \( \frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots \) all equal 1.
- \( \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1} \) are equivalent to 1, 2, 3, and 4, respectively
- \( \frac{2}{1}, \frac{4}{2}, \frac{6}{3} \) all equal 2.

Understanding these patterns helps students locate fractions on the number line.

USING REASONING

Students might use their knowledge about the number-line position of some fractions to help them reason about the position of other fractions. For example, they might know that \( \frac{4}{2} \) equals 2. Using this knowledge, students can reason that \( \frac{5}{2} \) is equal to 2 \( \frac{1}{2} \).

After students have placed the fraction cards on the number line, they may invite another group to check their work, or they can use Fra6.BLM3: Fraction Number Line to verify that fraction cards have been placed in their correct positions.

As students do the activity, ask them questions to help them to think about the methods they use to place the fractions on the number line:
- “What fraction did you create using the numbers on the number cubes?”
- “Where would this fraction be located on the number line?”
- “How do you know that this is the location of this fraction?”
- “How do you know that this fraction is greater than 1?”

Have students repeat the activity a few times. The last time they do the activity, ask students to print the fractions directly on the number line.
REFLECTING AND CONNECTING

Bring the class together for a discussion about the activity. Ask some general questions:

• “Which fractions were easy to locate on the number line? Why?”
• “Which fractions were difficult to locate? What was difficult about deciding where the fractions would be located?”
• “What strategies did you use to determine the location of fractions on the number line?”

Provide an opportunity for a few groups to show number lines on which they recorded the fractions. Ask students to explain the strategies they used to determine the position of each fraction on the number line.

Help students to make some generalizations about fractions by asking the following questions:

• “Which fractions are equal to 1?” (1/1, 2/2, 3/3, …)
• “Why are these fractions equal to 1?” (There are enough fractional parts to make the whole – for example, 2 halves make 1.)
• “What do you observe about the numerator and the denominator in these fractions?” (They are the same number.)
• “What other fractions would be equal to 1?”

Continue the discussion by asking questions about fractions equivalent to 2 and 3:

• “Which fractions are equal to 2?” (2/1, 4/2, 6/3, …)
• “Why are these fractions equal to 2?” (There are enough fractional parts to make 2 wholes – for example, 4 halves make 2.)
• “What do you observe about the numerator and the denominator in these fractions?” (The numerator is double the denominator.)
• “What other fractions would be equal to 2?”
• “Why are 3/1 and 6/2 equal to 3?”
• “What other fractions would be equal to 3?”

Pose similar questions about fractions that are equal to 4 to reinforce the patterns students observe.

Discuss proper and improper fractions. Ask:

• “How can you tell, just by looking at a fraction, that it is greater than 1?” (The numerator is greater than the denominator.)
• “What term is used to name a fraction in which the numerator is greater than the denominator?” (improper fraction)
• “Why is an improper fraction greater than 1?” (There are more fractional parts than 1 whole – for example, in 4/3, there are 3 thirds, or 1 whole, and another 1/3.)
• “How can you tell, just by looking at a fraction, that it is less than 1?” (The numerator is less than the denominator.)
• “What term is used to name a fraction in which the numerator is less than the denominator?” (proper fraction)
• “Why is a proper fraction less than 1?” (There are not enough fractional parts to make a whole – for example, in 2/3, another third would be needed to make a whole.)
Provide pairs of students with two large sheets of paper. Instruct pairs to work together to create two posters, entitled “Proper Fractions” and “Improper Fractions”, that explain the meaning of each type of fraction. Encourage students to use diagrams and words to clarify the terms.

Post the completed posters. Discuss and compare the ways in which students presented their ideas.

ADAPTATIONS/EXTENSIONS

Encourage students who have difficulty locating the position of fractions on the number line to use fraction models (e.g., fraction circles, diagrams, Fra6.BLM4: Fraction Circles) to help them think about the size of the fractions and their proximity to whole numbers. Some students may benefit from a version of the activity in which they consider the position of fewer fractions on a shorter number line. For this version of the activity, create number cubes with only the numbers 1, 2, 3 (each number printed twice on a cube) and have students work with a 0–3 number line.

Ask students who require a challenge to examine Fra6.BLM3: Fraction Number Line and to explain the fraction arrangements on the number line (e.g., the arrangement of fraction symbols in lower rows is more condensed than in higher rows). Students might determine that the fractional parts are increasingly smaller as they move from the top to the bottom row (e.g., fifths are smaller than fourths). Since the fractional parts in lower rows are smaller, more of them are required to make a whole.

Students could also use Fra6.BLM3: Fraction Number Line to find equivalent fractions (i.e., fractions that occupy the same position on the number line) and to extrapolate other equivalent fractions.

HOME CONNECTION

The letter on Fra6.BLM5: Ask Me About Fractions encourages parents/guardians to ask their child about the fraction concepts being learned in class. Before sending home the letter, conduct a think-pair-share activity to help students prepare for the discussion about fractions with their parents/guardians. Pose the following questions, one at a time, and provide time for students to think about their answers. Then ask students to share their ideas with a partner:
• “What are proper fractions?”
• “What are improper fractions?”
• “What are equivalent fractions?”
• “Which fractions are equal to 1?”

LEARNING CONNECTION 1
Fractions Between Fractions

MATERIALS
• a variety of manipulatives for representing fractions (e.g., fraction circles, Cuisenaire rods, counters, square tiles)
Arrange students in pairs. Challenge students to identify fractions that are between 1/2 and 3/4. Encourage students to use manipulatives and drawings.

As a whole class, discuss the fractions that were found, and ask students to explain how they know that the fractions are between 1/2 and 3/4.

Repeat by having students identify fractions that are between:
• 1/4 and 1/2;
• 1/8 and 1/2;
• 1/3 and 7/8.

LEARNING CONNECTION 2
From Least to Greatest

MATERIALS
• sheets of paper (1 per pair of students)

Arrange students in pairs. Challenge pairs to record all possible fractions (proper and improper) using only 6, 7, 8, and 9 as numerators and denominators. Next, have students arrange the fractions from least to greatest.

Combine pairs of students to form groups of four. Ask the groups to compare their ordered lists and to explain the strategies they used to order the fractions.

LEARNING CONNECTION 3
What’s the Whole?

MATERIALS
• pattern blocks (several per pair of students)
• overhead transparency of Fra6.BLM6: Finding the Part/Finding the Whole
• overhead projector

Provide opportunities for students to reflect on the relationships between fractional parts and the whole. Give each pair of students several pattern blocks. Display an overhead transparency of Fra6.BLM6: Finding the Part/Finding the Whole, and instruct students to work with their partners to solve the two problems.

After they have solved the problems, ask a few students to share their solutions with the class and to explain their thinking.
LEARNING CONNECTION 4
Fractions in a Venn Diagram

MATERIALS
- Fra6.BLM7: Organizing Fractions in a Venn Diagram (1 per pair of students)
- a variety of manipulatives for representing fractions (e.g., fraction circles, Cuisenaire rods, counters, square tiles)

Provide each pair of students with a copy of Fra6.BLM7: Organizing Fractions in a Venn Diagram. Instruct students to make a list of all proper fractions that have a denominator of 2, 3, 4, and 5. Have them record the fractions in the appropriate sections of the Venn diagram. Encourage students to use manipulatives to model fractions, if necessary.

Have students explain how they identified the fractions for each section of the Venn diagram.

LEARNING CONNECTION 5
Whose Fraction Is Greater?

MATERIALS
- number cubes (1 number cube per pair of students)
- sheets of paper (1 per student)
- a variety of manipulatives for representing fractions (e.g., fraction circles, fraction rectangles, two-colour counters)

Arrange students in pairs. Have students prepare a game sheet by drawing the following structure on their paper:

```
[Drawn structure]
```

Reject Boxes

Explain the game:
- The goal of the game is to create a fraction that is greater than the fraction created by the other player.
- Players take turns rolling a number cube and recording the number shown on the number cube in one of the boxes on their game sheet.
- Players may use the number from the roll of the number cube to create the numerator or denominator of a fraction, or they may record it in one of the reject boxes.
- After players have filled all the boxes on their game sheet, they compare their fractions to determine which player created the greater fraction.
Have a variety of manipulatives (e.g., fraction circles, fraction rectangles, two-colour counters) available, and encourage students to use them to compare the fractions that they created.

After students have played the game a few times, discuss strategies that they used to create the greatest fraction possible.

eWORKSHOP CONNECTION
Visit www.eworkshop.on.ca for other instructional activities that focus on fraction concepts. On the home, click “Toolkit”. In the “Numeracy” section, find “Fractions (4 to 6)”, and then click the number to the right of it.
Number-Line Strip

Make a 1 to 6 number line. Cut out both sections. Glue the tab on the second section to the back of the first section.

Glue this tab to the back of the other section of the number line.
Fraction Circles
Ask Me About Fractions

Dear ________________,

I have been learning about fractions. I can tell you what I know about fractions, so ask me about:

• proper fractions;
• improper fractions;
• equivalent fractions;
• fractions that are equal to 1.

Thanks for letting me tell you what I know about fractions.

Sincerely,

_________________

Note to parent/guardian: The discussion about fractions provides an opportunity for your child to review what he or she has learned in math class. During the discussion, ask questions such as the following:

• How did you learn that idea in class?
• Can you draw a diagram that shows that idea?
• Is that idea easy or difficult for you to understand? Why?

Please sign this sheet if you were able to have a discussion about fractions with your child, and have him or her return it to class.

Thank you for discussing fractions with your child.

____________________________________

Signature of parent/guardian
Finding the Part/Finding the Whole

1. If the yellow hexagon pattern block is 1 whole, find:
   a) one half.
   b) four sixths.
   c) seven sixths.

2. What would the whole look like if the red trapezoid is:
   a) one third?
   b) three fourths?
   c) three halves?
Organizing Fractions in a Venn Diagram

Fractions > \( \frac{1}{2} \)

Fractions < 1

Fractions With a Denominator of 5
Number Sense and Numeration, Grades 4 to 6

Volume 6
Decimal Numbers

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

Ontario Education excellence for all

2006
Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.
Number Sense and Numeration, Grades 4 to 6

Volume 6
Decimal Numbers

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6
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INTRODUCTION

*Number Sense and Numeration, Grades 4 to 6* is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories behind good instruction that are elaborated on in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The guide comprises the following volumes:
- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

The present volume – Volume 6: Decimal Numbers – provides:
- a discussion of mathematical models and instructional strategies that support student understanding of decimal numbers;
- sample learning activities dealing with decimal numbers for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume also contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by “eLearning modules” that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pp. 37, 54, and 76).
Relating Mathematics Topics to the Big Ideas

The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the “big ideas” of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005, p. 4)

In planning mathematics instruction, teachers generally develop learning activities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- operational sense
- proportional reasoning
- relationships

Each of the big ideas is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful...
for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

**Problem Solving:** Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

**Reasoning and Proving:** The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

**Reflecting:** Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

**Selecting Tools and Computational Strategies:** Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.

**Connecting:** The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping
students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

**Representing:** The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students’ own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

**Communicating:** Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

**Addressing the Needs of Junior Learners**

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners. The following table outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.
### Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
</table>
| Intellectual        | Generally, students in the junior grades:  
  • prefer active learning experiences that allow them to interact with their peers;  
  • are curious about the world around them;  
  • are at a concrete operational stage of development, and are often not ready to think abstractly;  
  • enjoy and understand the subtleties of humour. | The mathematics program should provide:  
  • learning experiences that allow students to actively explore and construct mathematical ideas;  
  • learning situations that involve the use of concrete materials;  
  • opportunities for students to see that mathematics is practical and important in their daily lives;  
  • enjoyable activities that stimulate curiosity and interest;  
  • tasks that challenge students to reason and think deeply about mathematical ideas. |
| Physical            | Generally, students in the junior grades:  
  • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys);  
  • are concerned about body image;  
  • are active and energetic;  
  • display wide variations in physical development and maturity. | The mathematics program should provide:  
  • opportunities for physical movement and hands-on learning;  
  • a classroom that is safe and physically appealing. |
| Psychological        | Generally, students in the junior grades:  
  • are less reliant on praise but still respond well to positive feedback;  
  • accept greater responsibility for their actions and work;  
  • are influenced by their peer groups. | The mathematics program should provide:  
  • ongoing feedback on students’ learning and progress;  
  • an environment in which students can take risks without fear of ridicule;  
  • opportunities for students to accept responsibility for their work;  
  • a classroom climate that supports diversity and encourages all members to work cooperatively. |
| Social              | Generally, students in the junior grades:  
  • are less egocentric, yet require individual attention;  
  • can be volatile and changeable in regard to friendship, yet want to be part of a social group;  
  • can be talkative;  
  • are more tentative and unsure of themselves;  
  • mature socially at different rates. | The mathematics program should provide:  
  • opportunities to work with others in a variety of groupings (pairs, small groups, large group);  
  • opportunities to discuss mathematical ideas;  
  • clear expectations of what is acceptable social behaviour;  
  • learning activities that involve all students regardless of ability. |
Characteristics of Junior Learners and Implications for Instruction

<table>
<thead>
<tr>
<th>Area of Development</th>
<th>Characteristics of Junior Learners</th>
<th>Implications for Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moral and ethical development</td>
<td>Generally, students in the junior grades:</td>
<td>The mathematics program should provide:</td>
</tr>
<tr>
<td></td>
<td>• develop a strong sense of justice and fairness;</td>
<td>• learning experiences that provide equitable opportunities for participation by all students;</td>
</tr>
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<td></td>
<td>• experiment with challenging the norm and ask “why” questions;</td>
<td>• an environment in which all ideas are valued;</td>
</tr>
<tr>
<td></td>
<td>• begin to consider others’ points of view.</td>
<td>• opportunities for students to share their own ideas and evaluate the ideas of others.</td>
</tr>
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</table>

(Adapted, with permission, from Making Math Happen in the Junior Grades. Elementary Teachers’ Federation of Ontario, 2004.)
LEARNING ABOUT DECIMAL NUMBERS IN THE JUNIOR GRADES

Introduction
Comprehending decimal numbers is an important development in students’ understanding of number. However, a deep understanding of decimal numbers can develop only when students have opportunities to explore decimal concepts concretely and pictorially, and to relate them to whole numbers and fractions.

PRIOR LEARNING
The development of whole number and fraction concepts in the primary grades contributes to students’ understanding of decimal numbers. Specifically, students in the primary grade learn that:

• our number system is based on groupings of 10 – 10 ones make a ten, 10 tens make a hundred, 10 hundreds make a thousand, and so on;
• fractions represent equal parts of a whole;
• a whole, divided into 10 equal parts, results in tenths.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES
Instruction that focuses on meaning, rather than on symbols and abstract rules, helps students understand decimal numbers and how they can be used in meaningful ways. In the junior grades, students gradually come to understand the quantity relationships of decimals to thousandths, relate fractions to decimals and percents, and perform operations with decimals to thousandths and beyond.

Developing a representational meaning for decimal numbers depends on an understanding of the base ten number system, but developing a quantity understanding of decimals depends on developing fraction sense. Students learn that fractions are parts of a whole – a convention developed to describe quantities less than one. This prior knowledge also helps students understand that decimals are numbers less than one whole. It is important to give students opportunities to determine for themselves the connections between decimals and fractions.
with denominators of 10 and 100. That understanding can then be developed with other fractions (i.e., with denominators of 2, 4, 5, 20, 25, and 50).

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to decimal numbers, listed in the following table.

### Curriculum Expectations Related to Decimal Numbers, Grades 4, 5, and 6

<table>
<thead>
<tr>
<th>Overall Expectations</th>
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<td>By the end of Grade 5, students will:</td>
<td>By the end of Grade 6, students will:</td>
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<td>• read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers;</td>
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<td>• demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts;</td>
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<td>• solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.</td>
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<td>• demonstrate an understanding of relationships involving percent, ratio, and unit rate.</td>
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### Overall Expectations

- By the end of Grade 4, students will:
  - read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100;
  - demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts;
  - solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.

- By the end of Grade 5, students will:
  - read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers;
  - demonstrate an understanding of magnitude by counting forward and backwards by 0.01;
  - solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.

- By the end of Grade 6, students will:
  - read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;
  - solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;
  - demonstrate an understanding of relationships involving percent, ratio, and unit rate.

### Specific Expectations

- Overall Expectations
  - read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100;
  - demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts;
  - solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.

- Overall Expectations
  - read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers;
  - demonstrate an understanding of magnitude by counting forward and backwards by 0.01;
  - solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.

- Overall Expectations
  - read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;
  - solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;
  - demonstrate an understanding of relationships involving percent, ratio, and unit rate.

### Specific Expectations

- Overall Expectations
  - read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100;
  - demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts;
  - solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.

- Overall Expectations
  - read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers;
  - demonstrate an understanding of magnitude by counting forward and backwards by 0.01;
  - solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.

- Overall Expectations
  - read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;
  - solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;
  - demonstrate an understanding of relationships involving percent, ratio, and unit rate.
The sections that follow offer teachers strategies and content knowledge to address these expectations in the junior grades while helping students develop an understanding of decimals. Teachers can facilitate this understanding by helping students to:

- relate fractions and decimal numbers;
- compare and order decimal numbers;
- explore various strategies for decimal-number computations.

### Relating Fractions and Decimal Numbers

Although adults quickly recognize that 0.5 and 1/2 are simply different representations of the same quantity, children have difficulty connecting the two different systems – fractional representation and decimal representation. It is especially difficult for children to make the connection when they are merely told that the two representations are “the same thing”.

Teachers in the junior grades should strive to see and explain that both decimal numbers and fractions represent the same concepts. This involves more than simply pointing out to students that a particular fraction and its corresponding decimal represent the same quantity – it involves modelling base ten fractions, exploring and expanding the base ten number system, and making connections between the two systems.
MODELLING BASE TEN FRACTIONS

Students need time to investigate base ten fractions, which are fractions that have a denominator of 10, 100, 1000, and so on. Both area models and length models can be used to explore these fractions. Although set models can also be used, they become impractical when working with anything other than tenths.

Base ten blocks or 10 \times 10 grids are useful models for working with tenths and hundredths.

**Strips-and-Squares Model**

It is important for students to understand that the large square represents one whole, 10 strips make one whole, and 100 smaller squares make one whole. Students can work with blank 10 \times 10 grids and shade sections in, or they can cut up coloured grids and place them on blank grids. Base ten blocks provide similar three-dimensional experiences.

When giving students problems that use the strips-and-squares model, the aim should be to develop concepts rather than rules. Some examples include:

- Fiona rolled a number cube 10 times, and 6 of those times an even number came up. Represent the number of even rolls as a fraction, and show it on a 10 \times 10 grid.
- Luis has 5 coins in his pocket. They total less than 1 dollar and more than 50 cents. How much money could he have? Write the amount as a fraction of a dollar, and shade the amount on a 10 \times 10 grid.
- What fraction of the grid at right is shaded? Write two different fractions to show the amount. How are the fractions related?
A hundredths wheel (or decimal wheel) also serves as an excellent area model for tenths and hundredths. A hundredths wheel is divided into 10 sections, each divided further into 10 equal intervals. When a slit is cut along one radius and two wheels of different colours are placed together, the model can be used to show decimals and fractions of less than one.

**Hundredths Wheel**

The wheel shows $0.28$ or $\frac{28}{100}$.

This model will be familiar to students, as many have seen “pies” divided into thirds, fourths, tenths, and so on. Ignoring the smaller graduations, the hundredths wheel is simply a tenths wheel.

Money also provides a model for hundredths that students are very familiar with. It is important for teachers to make connections to the knowledge students bring to the classroom, but it is also important to know the limits of a particular model. Investigating tenths with money is not as meaningful for students, since in everyday language we rarely refer to 6 dimes as “six tenths of a dollar”. Money amounts are usually represented to hundredths, but very rarely to one decimal place or three decimal places.

Length models are also useful for investigating base ten fractions. Paper strips can be divided into tenths, and metre sticks show both tenths (decimetres) and hundredths (centimetres). These concrete models transfer well to semi-concrete models, like number lines drawn with 10 or 100 divisions, and help students make quantity comparisons between decimals and base ten fractions.

**Metre Stick Showing Tenths and Hundredths of the Whole**
"Where would you place 0.6 on this number line? How about 0.06?"

I know 0.6 is 6/10, and the number line is divided into tenths (the larger lines), so 0.6 is the sixth large line. Each tenth is divided into 10 smaller pieces, so each of those is one hundredth, and 6 of the smaller lines are 0.06.

The advantage of each of the models presented so far is that the whole remains unchanged—it is simply divided into smaller pieces to represent hundredths.

The metre stick is an excellent model for thousandths when it is marked with millimetre increments. The length of the whole does not change, and students can see that each interval can be further subdivided (decimetres into centimetres, centimetres into millimetres) while the whole always stays the same.

Although area and three-dimensional models can also be used to represent thousandths, teachers should note that "redefining a whole" can be very confusing for students. Activities with base ten blocks that frequently redefine the whole should be pursued only with students who have a firm grasp of the concept of tenths and hundredths.

When representing thousandths with base ten blocks, students must define the large cube as one. (With whole numbers, the cube represented 1000.) With the large cube as one, flats become tenths, rods become hundredths, and units become thousandths. Although this three-dimensional model offers powerful learning opportunities, students should not be asked to redefine wholes in this way before having many rich experiences with base ten fractions.
Illustrated below is an easier-to-understand area model for thousandths, in which 10 × 10 grids are joined into a group of 10 to form a new whole.

**Area Model Showing Thousandths of the Whole**

![Area Model](image)

Although this two-dimensional model calls for the creation of a new whole – previously, with tenths and hundredths, the 10 × 10 grid was the whole – students readily can see that the model has grown larger to show the new whole. (Students have more difficulty understanding the model when using base ten blocks because the whole does not grow larger – the blocks are merely re-labelled when the whole changes.)

**EXPANDING THE BASE TEN NUMBER SYSTEM**

Many of the difficulties students have with decimal numbers stem from the fact that decimals are primarily taught as an extension of the place-value system. Understanding how fractional amounts can be represented as decimals in the base ten number system is a key junior-grade concept.

In the primary grades, students learn that the idea of “ten makes one” is crucial to our number system. Ten ones make a ten; 10 tens make a hundred, and so on. Students in the junior grades extend this idea to larger numbers, like hundred thousands and millions. They may find it more difficult to extend this concept to numbers of less than one without multiple experiences. Although students may have an understanding of whole numbers (ones, tens, hundreds, ...), they may misunderstand the pattern of tens to the right of the decimal numbers and think of the first decimal place as oneths, the next place as tenths, and so on. Also, they may have difficulty recognizing that a decimal number such as 0.234 is both 2 tenths, 3 hundredths, 4 thousandths, as well as 234 thousandths.
Area models are effective for demonstrating that ten-makes-one also works “going the other way”. Base ten blocks are a three-dimensional representation of the strips-and-squares area model, which is shown below.

**Area Model Showing Strips and Squares**

Students’ initial experiences with this model involve moving to the **left**: ten squares make one strip; ten of those strips make a bigger square; and so on. Each new region formed has a new name and its own unique place in the place-value chart. Ten ones make 1 ten; 10 tens make 1 hundred; 10 hundreds make 1 thousand; and so on.

Teachers can build on this experience by having students investigate “going the other way”, which involves moving to the **right**. What happens if you take a square and divide it into ten equal strips? And what if you take one of those strips and divide it into ten smaller squares? Could you ever reach the smallest strip or square, or the largest strip or square?

Ultimately students should learn that this series involving ten-makes-one and one-makes-ten extends infinitely in both directions, and that the “pieces” formed when the whole is broken into squares or strips are special fractions (base ten fractions) – each with its own place in the place-value system.

The **decimal point** is a special symbol that separates the position of the whole-number units on the left from the position of the fractional units on the right. The value to the right of the decimal point is 1/10, which is the value of that place; the value two places to the right of the decimal point is 1/100, which is the value of that place; and so on.

Teachers can help students develop an understanding of the decimal-number system by connecting to the understandings that students have about whole numbers.

**Example 1: Students Read and Write Number Patterns**

Have students read and write numbers as follows:

- 222 000 two hundred twenty-two thousands
- 22 200 two hundred twenty-two hundreds
- 2220 two hundred twenty-two tens
- 222 two hundred twenty-two ones
Continue the pattern with decimals:

- 22.2 two hundred twenty-two tenths
- 2.22 two hundred twenty-two hundredths
- 0.222 two hundred twenty-two thousandths

Example 2: Using Different Number Forms

\[ 76 = 70 + 6 \]
\[ 425 = 400 + 20 + 5 \quad \text{OR} \]
\[ 4 \text{ hundreds} + 2 \text{ tens} + 5 \text{ ones} \quad \text{OR} \]
\[ 4 \text{ hundreds} + 1 \text{ ten} + 15 \text{ ones} \quad \text{OR} \]
\[ 3 \text{ hundreds} + 12 \text{ tens} + 5 \text{ ones} \]

Extend to decimals:

\[ 0.56 = 0.5 + 0.06 \quad \text{OR} \]
\[ 5 \text{ tenths} + 6 \text{ hundredths} \]

\[ 7.38 = 7 + 0.3 + 0.08 \quad \text{OR} \]
\[ 7 + 3 \text{ tenths} + 8 \text{ hundredths} \quad \text{OR} \]
\[ 7 + 2 \text{ tenths} + 18 \text{ hundredths} \quad \text{OR} \]
\[ 6 + 13 \text{ tenths} + 8 \text{ hundredths} \]

Activities like those in the examples not only use patterning to develop concepts, but also encourage students to think about how numbers greater than one can be represented using different base ten fractions. For example, reading 22.2 as “two hundred twenty-two tenths” requires students to think about how many tenths there are in 2 (20), and how many tenths there are in 20 (200).

**CONNECTING DECIMALS AND FRACTIONS**

Students in the junior grades begin to work flexibly between some of the different representations for rational numbers. For example, if asked to compare 3/4 and 4/5, one strategy is to convert the fractions to decimal numbers. 3/4 is 0.75 (a commonly known decimal linked to money), and 4/5 can be thought of as 8/10, which converts to 0.8. These fractions, represented as decimal numbers, can now be easily compared.

When connecting the two different representations, it is important for teachers to help students make a conceptual connection rather than a procedural one. Conversion between both representations can (unfortunately) be taught in a very rote manner – “Find an equivalent fraction with tenths or hundredths as the denominator, and then write the numerator after the decimal point.” This instruction will do little to help students understand that decimals are fractions. Instead, students need to learn that fractions can be turned into decimals, and vice versa.

Activities should offer students opportunities to use concrete base ten models to represent fractions as decimals and decimals as fractions. For example, consider the following.

“Use a metre stick to represent 2/5 as a decimal.”
Students will have used this model to explore base ten fractions before being given this problem. Here is one student’s solution:

![Diagram showing 40 cm or 4 tenths = 0.4]

I used the metre stick as one whole, or 1. To figure out where \(\frac{2}{5}\) was, I divided the stick into 5 equal parts. 100 cm ÷ 5 = 20 cm, so \(\frac{2}{5}\) is at the 40 cm mark. 40 cm is 4 tenths of the metre stick, or 0.4. So \(\frac{2}{5}\) can be written as 0.4.

Similar activities using strips and squares, or base ten blocks, also help students to make connections between the representations.

It is important for students to experience a range of problem types when making connections between decimals and fractions. Sample problem types with numbers less than one include:

- given the fraction, write the decimal equivalent;
- given the decimal, write the fraction equivalent;
- given a fraction and decimal, determine if they are equivalent representations.

Also, problems should involve determining fractional amounts greater than one (e.g., write the decimal equivalent for \(2 \frac{72}{100}\)).

**Comparing and Ordering Decimal Numbers**

Shopping and measuring are real-life activities in which decimal numbers often need to be compared or ordered. Learning activities in which students compare and order decimal numbers not only develop practical skills, but also help to deepen students’ understanding of place value in decimal numbers. Students can compare and order decimal numbers using models and reasoning strategies.

**USING AREA MODELS OR BASE TEN BLOCKS**

Concrete materials, such as fraction circles, 10×10 grids, and base ten blocks, allow students to compare and order decimal numbers. Models provide visual representations that show the relative size of the decimal numbers.
To compare 0.3 and 0.5 using base ten blocks, for example, the rod could be used to represent the whole, and the small cubes to represent tenths.

\[0.3 \text{ < } 0.5\]

To compare 0.4 and 0.06, a strips-and-squares model could be used. A large square would become the whole; the strip, one tenth; and the smaller square, one hundredth.

\[0.4 \text{ > } 0.06\]

10×10 grids allow students to colour or shade in strips and squares to compare decimal numbers. For example, students can use a 10×10 grid to compare 0.6 and 0.56:

\[0.6 \text{ > } 0.56\]

**USING LENGTH MODELS**

A metre stick is an excellent model for comparing decimal numbers. To compare 0.56 and 0.8, for example, students can use the centimetre and decimetre increments to locate each number on the metre stick. Each centimetre is 1/100 or 0.01 of the whole length. Fifty-six hundredths, or 0.56, is at the 56 cm mark; and eight tenths, or 0.8, is 8 dm or 80 cm.
Locating numbers on a number line extends the physical model and connects to students’ prior learning with whole numbers. Before comparing and ordering decimals, students should have meaningful experiences with locating decimals on a number line. Some sample problems are:

- Draw a number line that starts at 0 and ends at 1. Where would you put 0.782? Why?
- On a number line that extends from 3 to 5, locate 4.25, and give reasons for your choice.
- 2.5 is halfway between 1 and 4. What number is halfway between 1 and 2.5? Use a number line and explain your reasoning.

Partial number lines can be used to order decimals as well. For example, to order 2.46, 2.15, and 2.6, students could draw a number line that extends from 2 to 3, then mark the tenths between 2 and 3, and then locate the decimal numbers.

To help students visualize hundredths and beyond, sections of the number line can be “blown up” or enlarged to show smaller increments.

Blowing up a section of this number line will allow students to model thousandths in a similar manner.

**USING REASONING STRATEGIES**

After students have had opportunities to compare decimal numbers using models and number lines, they can compare decimals using reasoning strategies that are based on their understanding of place value.

For example, to compare 3.45 and 3.7, students observe that both numbers have the same number of ones (3), and that there are 7 tenths in 3.7, but only 4 tenths in 3.45. Therefore, 3.45 is less than 3.7, even though there are more digits in 3.45. Teachers need to be cautious that this type of reasoning does not become overly procedural, however. Consider, on the following page, how the student is comparing 15.15 and 15.9, and demonstrates only a procedural knowledge of comparing decimals:
Students gain little understanding of quantity if they compare decimals by looking from digit to digit. Students should apply whole-number reasoning strategies and use benchmarks. For example, when comparing 15.15 and 15.9, students should recognize that 15.15 is a little bigger than 15, and that 15.9 is almost 16, so 15.15 is the smaller number.

1/2 or 0.5 is an important benchmark as well. When asked to order 6.52, 5.9, 6.48, 6.23, and 6.7, students can use 6.5 as the halfway point between 6 and 7. Students should recognize that 5.9, 6.23, and 6.48 are all less than 6.5, and 6.52 and 6.7 are greater than 6.5.

**Strategies for Decimal-Number Computations**

Strategies for decimal-number computations can be found in Volume 2: Addition and Subtraction, Volume 3: Multiplication, and Volume 4: Division.

**A Summary of General Instructional Strategies**

Students in the junior grades benefit from the following instructional strategies:

• representing decimal numbers using a variety of models, and explaining the relationship between the decimal parts and the whole;

• discussing and demonstrating base ten relationships in whole numbers and decimal numbers (e.g., 10 ones make ten, 10 tenths make one, 10 hundredths make a tenth);

• using models to relate fractions and decimal numbers (e.g., using fraction strips to show that 2/10 = 0.2);

• comparing and ordering decimal numbers using models, number lines, and reasoning strategies;

• investigating various strategies for computing with decimal numbers, including mental and paper-and-pencil methods.

The Grades 4-6 Decimal Numbers module at www.eworkshop.on.ca provides additional information on developing decimal concepts with students. The module also contains a variety of learning activities and teaching resources.
REFERENCES


Learning Activities for Decimal Numbers

Introduction

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to decimal numbers. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts in order to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or activity.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students’ understanding of mathematical concepts.
HOME CONNECTION: This section is addressed to parents or guardians, and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.
Grade 4 Learning Activity
Decimal Game

OVERVIEW
In this learning activity, students play a game in which they shade tenths of fraction strips and express the shaded parts as fractions and decimal numbers. The activity helps students to understand decimal quantities and the meaning of decimal-number notation.

BIG IDEAS
This learning activity focuses on the following big ideas:
Quantity: Students explore decimal quantities by shading fraction strips divided into tenths.
Relationships: This learning activity allows students to see relationships between fractions and decimal numbers (e.g., both number forms can be used to represent tenths).
Representation: Students learn about decimal-number representations (e.g., the role of the decimal point). They also explore the idea that fractions and decimal numbers can represent the same quantity (e.g., $\frac{3}{10} = 0.3$).

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations.
Students will:
• represent, compare, and order decimal numbers to tenths, using a variety of tools (e.g., concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings) and using standard decimal notation;
• determine and explain, through investigation, the relationship between fractions (i.e., halves, fifths, tenths) and decimals to tenths, using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., decompose $\frac{2}{5}$ into $\frac{4}{10}$ by dividing each fifth into two equal parts to show that $\frac{2}{5}$ can be represented as 0.4).
These specific expectations contribute to the development of the following overall expectation.
Students will:
• read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100.
ABOVE THE LEARNING ACTIVITY

MATERIALS
- Dec4.BLM1: Fraction Strip Divided Into Tenths
- chart paper
- markers
- overhead transparency of Dec4.BLM2: Fraction Strips
- overhead projector
- Dec4.BLM3: Tenths Game Board (1 per student)
- ten-sided number cube (1 number cube per pair of students), or alternatively, spinners made from Dec4.BLM4: Ten-Section Spinner, a paper clip, and a pencil (1 per pair of students)
- a variety of manipulatives for representing tenths (e.g., fraction circles, counters, square tiles)
- Dec4.BLM5: Cover the Tenths Game (1 per student)

MATH LANGUAGE
- tenths
- fraction
- decimal point
- representation
- decimal number
- greater than
- fraction
- less than

INSTRUCTIONAL SEQUENCING
Before starting this learning activity, students should have had experience representing tenths as parts of whole objects and representing tenths using fraction notation (e.g., 4 tenths can be recorded as “4/10”). In this learning activity, students continue to explore the concept of tenths and learn that tenths can be represented as decimal numbers.

ABOUT THE MATH
In this learning activity, students review the concept of tenths as parts of a whole, and explore how tenths can be represented as fractions and as decimal numbers. The learning activity helps students to recognize that both notations (fraction and decimal number) represent the same quantity.

When students understand that tenths can be expressed as fractions and as decimals, they are able to recognize equivalent representations of the same number (e.g., 1/2 and 0.5), allowing them to choose the more useful representation in different situations. For example, it may be easier for some students to add 1/2 and 4/10 by thinking of them as 0.5 and 0.4.

GETTING STARTED
Show students Dec4.BLM1: Fraction Strip Divided Into Tenths, and ask: “If this strip represents one whole, what is each part called? How do you know?”
Ask students to explain different ways to represent “tenths”. For example, they might suggest using:

- concrete materials (e.g., snapping together 10 interlocking cubes in a row and recognizing that each cube is a tenth of the row);
- diagrams (e.g., drawing a shape, such as a circle, square, or rectangle, and dividing the shape into 10 equal parts);
- symbols (e.g., recording the fraction 1/10).

Record students’ ideas on chart paper or the board using diagrams, symbols, and words.

Refer to each recorded representation and ask the following questions:

- “In this representation (concrete material, diagram, symbol), what does the whole look like?”
- “How do you know that this part (interlocking cube, section of the rectangle, number) is one tenth of the whole?”
- “How could you show 2 tenths? 3 tenths? 10 tenths?”
- “How could you show 11 tenths using interlocking cubes? A diagram? A fraction?”

Explain that decimal numbers can also represent tenths. Record “0.1” on the board, and explain that this decimal number is read as “one tenth”. Discuss the following ideas:

- The decimal point separates the whole-number part of the number from the decimal-number part.
- The zero to the left of the decimal point shows that there are no ones.
- The place-value column to the right of the decimal point tells the number of tenths. For example, in 0.1, there is 1 tenth.

Ask students to suggest places where they have seen decimal points (e.g., prices, measurements, sports statistics).

Display an overhead transparency of **Decimal BLM: Fraction Strips**. To begin, show only the fraction strip with 4 tenths shaded. Ask:

- “If the strip represents 1 whole, what part of the strip is shaded?”
- “How do you know that 4 tenths is less than 1?”
- “How would you record 4 tenths as a fraction?”
- “How would you record 4 tenths as a decimal number?”

Record both the fraction (4/10) and the decimal number (0.4) below the fraction strip on the overhead transparency.

Continue the discussion by having students describe the other fraction strips (6 tenths, 8 tenths) on the overhead transparency. Ask them to give both the fraction and the decimal representations for each fraction strip. Label the fraction strips accordingly.
WORKING ON IT

Arrange students in pairs. Provide each student with a copy of Dec4.BLM3: Tenths Game Board. Give each pair of students a ten-sided number cube. (Alternatively, have them use Dec4.BLM4: Ten-Section Spinner). Explain that students will play a game that will allow them to represent tenths in different ways.

Explain the game procedures:
• The first player rolls the number cube. Whatever number is rolled, the student shades in that many sections of a fraction strip on his or her copy of Dec4.BLM3: Tenths Game Board. For example, if a 7 is rolled, the student shades in 7 sections of the strip. The student announces the number that is shaded ("seven tenths") and then records the number as a decimal number and as a fraction, below the shaded strip.
• The second player rolls the number cube and completes a section of his or her game board.
• Players continue to take turns.
• If a player rolls a number that he or she has already rolled, that player does not shade in a fraction strip.
• Players should check each other’s game board as they are playing, to make sure that the numbers are being written correctly.
• The first player to complete his or her game board, by shading fraction strips and recording corresponding fraction and decimal numbers for one tenth through to ten tenths, wins the game.

Observe students while they play the game. Note whether they record appropriate fraction and decimal representations for each shaded fraction strip. Observe, as well, what students do when they roll a 10. (Do students have difficulty grasping that 1.0 is the same as ten tenths?)

Ask students questions such as the following:
• "What number does this fraction strip show?"
• "How can you record this number as a fraction? How can you record the number as a decimal number?"
• "How do you know that this fraction and this decimal number represent the same quantity?"
• "How can you represent ten tenths on the fraction strip? As a fraction? As a decimal number?"
• "How do you know what numbers you still need to roll?"

REFLECTING AND CONNECTING

Reconvene students after the game. Talk to them about how they represented tenths using diagrams (fraction strips), as decimal numbers, and as fractions. As an example, show a fraction strip with 6 tenths shaded in, and ask students to explain two ways to record the number. Record "6/10" and "0.6" on the board.

Ask students to explain what they learned about decimal numbers when they played the game. Record students’ ideas on the board or chart paper. For example, students might explain the following ideas:
• Both 4/10 and 0.4 represent 4 tenths.
• The number of tenths is recorded to the right of the decimal point.
• If there is no whole-number part (i.e., the number is less than 1), 0 is recorded to the left of the decimal point.
• 10 tenths is the same as 1 whole.
• 10 tenths is recorded as 1.0.

Initiate a discussion about decimal-number quantities. Write “0.5” and “0.8” on the board, and ask students to identify the greater number. Have students explain why 0.8 is greater than 0.5. Listen for student responses like, “Because 8 is bigger than 5.” Write 5.0 and 0.8 on the board as an illustration of why this explanation is not sufficient. Encourage students to draw a diagram (e.g., showing fraction strips) on the board to demonstrate that 0.8 is a greater quantity than 0.5.

As well, ask students to name numbers that are greater than 0.7 but less than 1.0. Have them model 0.8 and 0.9 using fraction-strip diagrams.

During the discussion, clearly model the use of the decimal point and its role in separating the part of the number that is a whole number from the part of the number that is less than one whole (decimal numbers). You may want to extend the discussion to include numbers like 2 and 6 tenths (2.6), or 3 and 6 tenths (3.6). Ask students to represent these numbers using manipulatives (e.g., fraction circles, counters, square tiles) or drawings.

ADAPTATIONS/EXTENSIONS

Students need a strong understanding of fractions before they will be able to grasp the concept that fractions and decimal numbers can represent the same quantity. Simplify the game by having students shade in the fraction strips according to the number shown on the number cube, and have them record only the fraction that represents the shaded portion of the fraction strip.

For students requiring a greater challenge, provide them with two six-sided number cubes instead of one ten-sided number cube. To complete their game card, students roll both number cubes, and then choose to add or subtract the numbers rolled to determine how many tenths to shade.
ASSESSMENT

Observe students as they play the game, and assess how well they:
• shade in fraction strips according to the number shown on the number cube;
• identify the shaded part of fraction strips (e.g., 3 shaded spaces shows “three tenths”);
• record fractions and decimal numbers that represent the shaded part of fraction strips;
• explain that a fraction and a decimal number represent the same quantity.

HOME CONNECTION

Send home copies of Dec4.BLMS: Cover the Tenths Game. The game provides an opportunity for students and their parents/guardians to represent tenths as fractions and as decimal numbers.

LEARNING CONNECTION 1

Decimal Numbers Using Base Ten Blocks

MATERIALS
• overhead base ten blocks (flats, rods, small cubes) or regular base ten blocks (flats, rods, small cubes)
• overhead projector

This activity reinforces students’ understanding of decimal-number quantities.

Show students a rod from a set of base ten blocks (use overhead blocks, if available) using an overhead projector. Ask: “What number does this rod represent?” Students will likely answer “10”. Ask: “Does the rod have to be 10? Could it represent 1?” Explain that the rod, for the purposes of the activity, will represent 1.

Display a flat using the overhead projector, and ask students, “If the rod represents 1, what number does the flat represent?” Ask students to explain how they know that the flat represents 10.

Next, show a small cube and ask students, “If the rod represents 1, what does the small cube represent?” Ask students to explain how they know that the small cube represents one tenth. Reinforce the idea that the rod is made up of 10 small cubes, so each small cube is one tenth of the rod.

Place two rods and a small cube on the overhead projector, and ask students to identify the number that is represented. On the board, record “two and one tenth”, “2 1/10”, and “2.1”. Discuss how all three notations represent the quantity shown by the base ten blocks.

Repeat with several other numbers so that students are comfortable representing numbers using a place-value chart.
LEARNING CONNECTION 2
Decimal Number Grab Bags

MATERIALS
- Dec4.BLM6: Place-Value Mat (1 per pair of students)
- Dec4.BLM7: Decimal Number Grab Bag Recording Sheet (1 per pair of students)
- Large paper bags containing base ten blocks – including 3 to 5 flats, 5 to 9 rods, and 10 to 20 small cubes (1 per pair of students)

Provide each pair of students with a copy of Dec4.BLM6: Place-Value Mat, a copy of Dec4.BLM7: Decimal Grab Bag Recording Sheet, and a large paper bag containing base ten blocks. Show that the bag contains base ten blocks, and explain that a rod represents 1, a flat represents 10, and a small cube represents 1 tenth.

Explain the activity:
- Pairs of students take turns “grabbing” (using both hands) a quantity of base ten blocks from the bag, and organizing the blocks on the place-value mat (trading 10 cubes for a rod, if necessary).
- Both students record a drawing, a fraction, and a decimal number to represent the number drawn from the bag.
- Students compare what they have recorded.
- Students return the materials to the bag after each turn.

As students work on the activity, ask questions such as the following:
- “What number did you grab from the bag?”
- “How can you represent this number using a drawing? A fraction? A decimal number?”
- “How does the place-value mat help you organize the number?”

There are several variations for the activity. For example, students could:
- compare each new number to the previous one by deciding if it is greater or less;
- challenge their partners to grab a number that is greater than or less than the previous number;
- add each new decimal number to the previous numbers.

LEARNING CONNECTION 3
Closest to Ten

MATERIALS
- Six-sided number cubes (2 per pair of students)
- Dec4.BLM8: Closest to Ten Recording Sheet (1 per student)
- A variety of manipulatives for representing decimal numbers (e.g., fraction circles divided into tenths, base ten blocks, counters)

This game provides an opportunity for students to add and subtract decimal numbers to tenths.

Have students play the game in pairs. To begin, one player rolls two number cubes and uses the numbers rolled to create a decimal number containing a whole-number digit and a tenths
digit (e.g., after rolling a 5 and a 3, a player can create either 5.3 or 3.5). The second player then does the same. Each player records the numbers he or she created beside “Roll 1” on the Dec. BLM: Closest to Ten Recording Sheet.

Players continue to create decimal numbers. Each time they record a number, players must either add the number to or subtract it from the previous number on the Dec. BLM: Closest to Ten Recording Sheet. The resulting sum or difference is recorded in the appropriate space.

After five rolls, the player whose final number is closest to 10 wins. (The number may be less than or greater than 10.)

Encourage students to use manipulatives (e.g., fraction circles divided into tenths, base ten blocks, counters) to help them add or subtract their numbers.

Reconvene the class after students have played the game a few times. Discuss the game by asking questions such as the following:

• “What strategies did you use to add decimal numbers?”
• “What strategies did you use to subtract decimal numbers?”
• “How did you decide whether to add or subtract two numbers?”
• “How did you figure out who won the game?”

Provide an opportunity for students to play the game again, so that they can try the strategies they learned about during the class discussion.

LEARNING CONNECTION 4
Counting Tenths

MATERIALS
• calculators (1 per student)
• overhead calculator, if available

Note: Check that calculators have the memory feature required for this activity. Enter + .1 and then press the = key repeatedly. The display should show 0.2, 0.3, 0.4, and so on. If you are using the TI-15 calculator, you will have to use the OP1 or OP2 buttons.

Counting by tenths helps to build an understanding of decimal quantity and can reinforce an understanding of the relationship between tenths and the whole.

Provide each student with a calculator. Instruct students to enter + .1 in the calculator and ask them to read the number (“one tenth”). Next, have students press the = key and read the number. Have them continue to press the = key repeatedly and read the number on their calculators each time.

When students reach 0.9, have them predict what their calculators will show when they press the = key. (Some students may predict that the calculators will show 0.10.) Have students check their predictions by pressing =, and discuss how 1 represents one whole.
Have students continue counting with their calculators (1.1, 1.2, 1.3, ...). When students reach 1.9, have them predict the next number before continuing to count. They can continue counting by tenths until they reach 3 or 4.

Conclude the counting activity by asking the following questions:
• “How many tenths did you add to get from 1 to 2?” (10)
• “How many tenths are there altogether in 2?” (20)
• “How many tenths would you need to add to make 1.6 (2.8, 3.1) appear on your calculator?”

eWORKSHOP CONNECTION
Visit www.eworkshop.on.ca for other instructional activities that focus on decimal concepts. On the homepage click “Toolkit”. In the “Numeracy” section, find “Decimal Numbers (4 to 6)”, and then click the number to the right of it.
Tenths Game Board

Fraction: _______________
Decimal Number: ___________

Fraction: _______________
Decimal Number: ___________

Fraction: _______________
Decimal Number: ___________

Fraction: _______________
Decimal Number: ___________

Fraction: _______________
Decimal Number: ___________

Fraction: _______________
Decimal Number: ___________

Fraction: _______________
Decimal Number: ___________

Fraction: _______________
Decimal Number: ___________

Fraction: _______________
Decimal Number: ___________
**Ten-Section Spinner**

Make a spinner using this page, a paper clip, and a pencil.
Cover the Tenths Game

Dear Parent/Guardian:

We have been learning that tenths can be written as fractions and as decimal numbers. For example, three tenths of the following rectangle is shaded.

Three tenths can be written as 3/10 and 0.3.

Here is a game to play with your child. You will need 10 small objects (e.g., buttons, coins, beans), a piece of paper, and a pencil.

• Players take turns covering spaces in the rectangle at the bottom of this page with small objects (one small object per space).

• At each turn, players decide whether to cover one, two, or three spaces.

• After placing the small objects in the spaces, players announce the total number of spaces that are covered and write the number of covered spaces as a fraction and as a decimal number on a piece of paper. For example, if 4 spaces are covered, the player would announce “4 tenths” and would record “4/10” and “0.4”.

• The player who finishes covering all 10 spaces announces “10 tenths is 1 whole” and records 10/10 and 1.0. This player wins the game.

After playing the game several times, ask your child to explain strategies for winning the game.

Thank you for doing this activity with your child.
Place-Value Mat

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
### Decimal Number Grab Bag Recording Sheet

<table>
<thead>
<tr>
<th>Drawing</th>
<th>Fraction</th>
<th>Decimal Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tens</td>
<td>Ones</td>
<td>Tenhs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>34 3/10</td>
</tr>
</tbody>
</table>
## Closest to Ten Recording Sheet

<table>
<thead>
<tr>
<th>Roll 1</th>
<th>Roll 2</th>
<th>Sum or Difference</th>
<th>Roll 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll 4</td>
<td>Roll 5</td>
<td>Final Sum or Difference</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Roll 1</th>
<th>Roll 2</th>
<th>Sum or Difference</th>
<th>Roll 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll 4</td>
<td>Roll 5</td>
<td>Final Sum or Difference</td>
<td></td>
</tr>
</tbody>
</table>
Grade 5 Learning Activity
Number Books

OVERVIEW
In this learning activity, students play a game in which they match cards showing partially shaded 10 × 10 grids with corresponding fraction and decimal cards. The game helps to develop students’ understanding of tenths and hundredths, and emphasizes the relationships between fractions and their equivalent decimal forms.

BIG IDEAS
This learning activity focuses on the following big ideas:

Quantity: In the Number Books game, students match fraction and decimal cards to quantities represented by partially shaded 10 × 10 grids.

Relationships: The game focuses on relationships between fractions and their equivalent decimal forms (e.g., 4/10 = 0.4; 25/100 = 0.25). It also helps students to recognize equivalencies between fractions (e.g., 3/10 = 30/100) and between decimal numbers (e.g., 0.3 = 0.30).

Representation: The game provides an opportunity for students to identify equivalent fraction-decimal number representations (e.g., 0.4 = 40/100 = 4/10 = 2/5).

Proportional reasoning: Students determine the relationship between simple fractions, fractions with a denominator of 10 or 100, and equivalent decimal numbers (e.g., 3/4 = 75/100 = 0.75).

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations.
Students will:
• demonstrate and explain the concept of equivalent fractions, using concrete materials (e.g., use fraction strips to show that 2/4 is equal to 4/8);
• demonstrate and explain equivalent representations of a decimal number, using concrete materials and drawings (e.g., use a base ten materials to show that three tenths [0.3] is equal to thirty hundredths [0.030]);
• determine and explain, through investigation using concrete materials, drawings, and calculators, the relationship between fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100) and their equivalent decimal forms (e.g., use a 10 × 10 grid to show that 2/5 = 40/100, which can also be represented as 0.4).
These specific expectations contribute to the development of the following overall expectation.

Students will:
• read, represent, compare, and order whole numbers to 100,000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers.

ABOUT THE LEARNING ACTIVITY

MATERIALS
• overhead transparency of Dec5.BLM1: Twenty-Five Hundredths
• overhead projector
• overhead marker
• overhead transparency of Dec5.BLM2: Sixty Hundredths
• Dec5.BLM3: 10 x 10 Grid
• Dec5.BLM4: 10 x 10 Grid Cards (1 set of 9 cut-out cards per pair of students)
• Dec5.BLM5a-c: Number Cards (1 set of 27 cut-out cards per pair of students)
• a few transparencies of Dec5.BLM3: 10 x 10 Grid
• Dec5.BLM6: Blank 10 x 10 Grid Cards
• Dec5.BLM7: Blank Cards
• Dec5.BLM8: Money Amounts as Decimal Numbers

MATH LANGUAGE
• tenths
• hundredths
• representation
• equal/equivalent

INSTRUCTIONAL SEQUENCING
Before starting this learning activity, students should have had experiences representing tenths as fractions (e.g., 4/10) and as decimal numbers (e.g., 0.4). This learning activity provides opportunities for students to extend their understanding of decimal numbers by exploring the meaning and representations of hundredths.

ABOUT THE MATH
Instructional activities with 10 x 10 grids help students to develop a sense of quantity about tenths and hundredths, and to grasp the relationship between one, tenths, and hundredths. The grid allows students to consider one column (or one row) as a tenth of the entire grid. The same row (or column) represents 10 hundredths of the grid.
The 10 × 10 grid also helps students recognize relationships between fractions and decimal numbers.

By dividing the grid into equal parts (e.g., into fifths, as illustrated in the following 10 × 10 grid), students can observe the relationship between decimal numbers and fractions with denominators other than 10 and 100.

The shaded part of the grid represents 1 tenth or 10 hundredths of the grid.

![Grid Image]

The 10 × 10 grid also helps students recognize relationships between fractions and decimal numbers.

By dividing the grid into equal parts (e.g., into fifths, as illustrated in the following 10 × 10 grid), students can observe the relationship between decimal numbers and fractions with denominators other than 10 and 100.

Four tenths (0.4) of the grid is shaded.

Two fifths (\(\frac{2}{5}\)) of the grid is shaded.

The focus in the following learning activity is on equal representations (e.g., the equivalence of 0.4, 40/100, 4/10, and 2/5). The notion that different numerical forms can represent the same quantity is a major development in students’ understanding about numbers.
GETTING STARTED

Display an overhead transparency of Dec5.BLM1: Twenty-Five Hundredths. Discuss how the grid has 10 rows with 10 squares in each row, and establish that there are 100 squares in the grid. Refer to the grid as a "10 × 10 grid" and emphasize the idea that each square is one hundredth of the grid.

Ask: "What are different ways to describe the part of the grid that is shaded?" Provide time for students to think about the question individually, and then have them share their ideas with a partner. After partners have discussed the question, invite a few students to share their ideas with the whole class.

Discuss the following ideas:
• Since the grid is divided into 100 squares, each square is 1 hundredth of the grid, and 25 hundredths are shaded. Discuss different ways of recording the number:
  – twenty-five hundredths (in words)
  – 0.25 (as a decimal number)
  – 25/100 (as a fraction with a denominator of 100)
• The grid can be divided into four equal parts. The shaded part is one fourth of the grid.
  To illustrate this idea, use an overhead marker to outline four equal parts on the grid. Ensure that students recognize that all four parts are equal in size. (All parts have 25 squares.)

Review the meaning of 1/4 – the denominator represents the number of equal parts, and the numerator represents the number of parts being considered (in this case, the shaded part). Emphasize the notion that 0.25, 25/100, and 1/4 are different ways to represent the same quantity.

Display an overhead transparency of Dec5.BLM2: Sixty Hundredths. Have students work with a partner to discuss and record different ways to represent the shaded part of the grid.

Ask a few students to share their work. Discuss how the shaded part of the grid can be represented using words (sixty hundredths), decimal numbers (0.60, 0.6), and fractions (60/100, 6/10, 3/5). Use an overhead marker to highlight areas of the grid in order to demonstrate that 0.60 (sixty hundredths) is equal to 0.6 (six tenths), and that 60/100 is equal to 6/10 and 3/5.

Dec5.BLM3: 10 × 10 Grid is a blank grid, which can be partially shaded if students require more experience in determining equivalent number representations.
WORKING ON IT

Tell students that they will be playing a game called Number Books. Explain that the goal of the game is to be the player who obtains the most “number books”. Explain that a complete number book consists of a $10 \times 10$ grid card and its matching fraction card, decimal card, and hundredths card.

Demonstrate the game by playing it with a volunteer.

- Each pair of players needs a deck of $10 \times 10$ grid cards (cut from Dec5.BLM4: 10 x 10 Grid Cards) and a deck of number cards (cut from Dec5.BLM5a-c: Number Cards). To prepare for the game, students draw six cards from the deck of $10 \times 10$ grid cards and display the cards, in a row, face up, between the players. One player deals four number cards to each player, and places the remaining number cards, face down, in a pile between the players.
- Players examine their hand of cards to find any number cards that match the displayed $10 \times 10$ cards. If they have one or more matching cards, they take turns placing the number cards face up, beside the corresponding $10 \times 10$ grid card. Players must lay down any matching cards they have in their hand. Players replace any cards they lay down by selecting cards from the pile of face-down cards; however, they may not play any of these cards until their next turn.
- If players are unable to lay down any cards during their turn, they place all four cards in their hand, face down, at the bottom of the number-card pile, and select another four number cards from the top of the pile. They may not, however, lay down any cards in the new hand until their next turn.
- A player who completes a book by laying down the fourth matching card may claim the four-card book and place it in front of him or her.
- When a four-card book is claimed, another $10 \times 10$ grid card is placed between the players. Players may lay down number cards beside this new card in an attempt to assemble another book.
- The game can be played for a specific amount of time (e.g., 20 minutes) or until all books have been assembled and claimed.

Have students play the game with a partner. As students play the game, ask them questions that focus on different ways to represent tenths and hundredths, and the relationship between these number forms:
- “What number cards will you need to complete this number book?”
- “Why do this fraction and this decimal number represent the same quantity?”
• “How do you know that 0.2 is the same as 20 hundredths?”
• “How do you know that this fraction (decimal number) matches the shaded part of this 10 × 10 grid?”

REFLECTING AND CONNECTING
Reconvene the class after students have played the game. Ask them to explain what they learned when they played the game. For example, students might discuss how the game helped them to recognize equivalent fractions and decimal numbers, and how it allowed them to better understand the quantity represented by fractions and decimal numbers.

Record 40/100, 2/5, and 0.4 on the board. Ask students to explain why all three numbers represent the same amount. Allow students to use an overhead transparency of Dec5.BLM3: 10 × 10 Grid and overhead markers to demonstrate the quantity (i.e., 40 hundredths, 2 fifths, 4 tenths) represented by all three numbers.

Discuss the equivalency of other numbers:
• 50/100, 1/2, 0.5
• 30/100, 3/10, 0.3
• 75/100, 3/4, 0.75
• 80/100, 4/5, 0.8

ADAPTATIONS/EXTENSIONS
Simplify the game for students who have difficulty matching equivalent fractions and decimal numbers. For example, students might play the game using only the 10 × 10 grid cards and the fraction cards, or the 10 × 10 grid cards and the decimal cards.

An easier version of the game can also involve 10 × 10 grids and matching number cards, including fractions with denominators of 100, decimal numbers (hundredths only), and hundredths cards.

Students or classroom volunteers can make the game cards using Dec5.BLM4: Blank 10 × 10 Grid Cards and Dec5.BLM7: Blank Cards.

Challenge students by asking them to use a 10 × 10 grid to prove that the following fractions are equivalent: 20/100, 10/50, 5/25, 4/20, 2/10, 1/5.
Have students use 10 × 10 grids to determine equivalent fractions for the following numbers:

- 25 hundredths
- 40 hundredths
- 60 hundredths
- 90 hundredths

**ASSESSMENT**

Observe students and converse with them as they play the game in order to assess how well they identify and explain equivalent forms of the same number (e.g., the equivalency of 0.5, 50/100, 5/10, 1/2).

Pose the following problem and write it on the board: “Seventy hundredths of Earth’s surface is covered with water. What part of Earth’s surface is covered by land?” Ask students to show a solution to the problem using fractions, decimal numbers, and diagrams, such as 10 × 10 grids.

Examine students’ responses to assess how well they:

- explain that 30 hundredths of Earth’s surface is covered by land;
- represent 30 hundredths using a fraction (30/100, 3/10), a decimal number (0.3), and a diagram (e.g., a shaded 10 × 10 grid).

**HOME CONNECTION**

Send home Dec5.BLM8: Money Amounts as Decimal Numbers. The game described in this Home Connection letter helps students to apply their understanding of decimal numbers to money representations.

**LEARNING CONNECTION 1**

*Using a Decimal Wheel*

**MATERIALS**

- copies of Dec5.BLM8: Hundreds Wheel, printed on two different colours of stiff paper
  - (1 copy of each colour per student)
- scissors (1 pair per student)
Note: Copies of Dec5.BLM9: Hundredths Wheel could be cut out and laminated ahead of time to increase their durability.

A hundredths wheel is a tool with which students can model decimal numbers (tenths and hundredths). Provide each student with a copy of Dec5.BLM9: Hundredths Wheel. Discuss how the circle is divided into ten equal sections (indicating tenths), and how it is marked with 100 equal intervals around the edge (indicating hundredths).

Distribute a second copy of Dec5.BLM9: Hundredths Wheel (printed on a different colour of paper), and instruct students to assemble the decimal wheel as follows.

• Have students cut out both circles, and then cut along the dotted line to the centre of each circle.
• Have them slip the circles together at the cut edges, so that the wheels can be rotated to show the different parts of a whole.

Begin by asking students to use their decimal wheels to show one half, then one fourth, then three fourths. Continue by having students model different decimal numbers you give orally (e.g., 3 tenths, 6 tenths, 60 hundredths, 67 hundredths, 6 hundredths). For each decimal number, ask students questions such as the following:

• “Is the decimal number shown on your wheel more or less than 1/2?”
• “Is the decimal number closer to 1/2 or 3/4?”
• “How far from one whole is the decimal number?”

Have students use their decimal wheels to model fractions and decimal numbers you record on the board (e.g., 7/10, 23/100, 3/10, 3/100). Have students discuss the following ideas by referring to their decimal wheels:

• fraction-decimal number equivalencies (e.g., 7/10 = 0.7);
• the composition of fractions with denominators of 100 as tenths and as hundredths (e.g., 23/100 = 2/10 + 3/100);
• the composition of decimal numbers as tenths and hundredths (e.g., 0.37 = 0.3 + 0.07).

Students can use decimal wheels to add and subtract decimal numbers. Begin by having students solve simple addition problems. For example, to calculate 0.5 + 0.3, students can show 5 tenths on the wheel, and then add 3 tenths by rotating the wheel for three more tenths. When students are familiar with the use of the decimal wheel for addition, they can use it to solve more complex problems (e.g., 0.2 + 0.28).
LEARNING CONNECTION 2

Decimal Golf

MATERIALS
- six-sided number cubes (2 per pair of students)
- Dec5.BLM10: Decimal Golf Score Cards (1 per pair of students)
- a variety of manipulatives for representing decimal numbers (e.g., 10 × 10 grids, hundredths wheels, metre sticks)

Provide each pair of students with two number cubes and a copy of Dec5.BLM10: Decimal Golf Score Cards. Ask students to examine the score card and draw their attention to the decimal number “par” that is given for each “hole number”.

Explain the game:
- Players take turns rolling two number cubes and using the digits on the number cubes to create a two-digit number that includes a decimal number (e.g., after rolling a 2 and a 5, a player could create 0.25, 0.52, 2.5, or 5.2). Players should try to create the number that is closest to par (the target number for each hole). Players record the number in the appropriate section of the score card.
- After both players have recorded their numbers for each hole, they determine which player recorded the number that is closer to par.
- The player with the number closer to par circles his or her number on the score card. If players tie, they both circle their numbers.
- Players may use manipulatives (e.g., 10 × 10 grids, hundredths wheels, metre sticks) to prove that their number is closer to par than the other player’s.
- The game continues until players have completed all nine holes. The winner is the player who has the most circled numbers.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on decimal concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Decimal Numbers (4 to 6)”, and then click the number to the right of it.
Twenty-Five Hundredths
Sixty Hundredths
10 x 10 Grid Cards
### Number Cards

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{2})</td>
<td>0.5</td>
<td>50</td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
<td>0.25</td>
<td>25</td>
</tr>
<tr>
<td>(\frac{3}{4})</td>
<td>0.75</td>
<td>75</td>
</tr>
</tbody>
</table>
### Number Cards

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{5}$</td>
<td>0.4</td>
<td>$\frac{3}{5}$</td>
<td>0.6</td>
</tr>
<tr>
<td>40 hundredths</td>
<td></td>
<td>60 hundredths</td>
<td></td>
</tr>
<tr>
<td>$\frac{4}{5}$</td>
<td>0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80 hundredths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction</td>
<td>Decimal</td>
<td>Hundredths</td>
<td></td>
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<td>---------------</td>
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<td>------------</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{10} )</td>
<td>0.3</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>( \frac{23}{100} )</td>
<td>0.23</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>( \frac{47}{100} )</td>
<td>0.47</td>
<td>47</td>
<td></td>
</tr>
</tbody>
</table>
Blank 10×10 Grid Cards
Money Amounts as Decimal Numbers

Dear Parent/Guardian:

We have been learning about decimal numbers in math class. Money amounts can be written as decimal numbers.

• 1¢ is 1 hundredth of a dollar. 1¢ can be written $0.01.
• 10¢ is 10 hundredths of a dollar. 10¢ can be written $0.10.
• 23¢ is 23 hundredths of a dollar. 23¢ can be written $0.23.

Play this game with your child. You will need a number cube (die) to play the game.

• Each player takes a turn rolling the number cube twice. The number on the first roll shows the number of dimes, and the number on the second roll shows the number of pennies. The player records the money amount as a decimal number. For example, by rolling a 3 and then a 5, a player has 3 dimes and 5 pennies, and records $0.35.
• After both players have recorded their decimal numbers, they decide which player has the greater money amount. The player with “more money” earns a point.
• The first player to earn 10 points wins the game.

Thank you for helping your child to understand decimal numbers.
Hundredths Wheel
## Decimal Golf Score Cards

<table>
<thead>
<tr>
<th>Hole Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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</thead>
<tbody>
<tr>
<td>Par</td>
<td>0.25</td>
<td>1.0</td>
<td>0.15</td>
<td>0.5</td>
<td>0.64</td>
<td>3.5</td>
<td>0.58</td>
<td>1.3</td>
<td>0.22</td>
</tr>
<tr>
<td>Player One:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Player Two:</td>
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</tbody>
</table>

<table>
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<th>Hole Number</th>
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</tbody>
</table>
Grade 6 Learning Activity
The Contest

OVERVIEW
In this learning activity, students are presented with a scenario about a contest involving three classes who donate to a food bank. Students are asked to justify the decision about the winning class, based on data about student participation expressed as a fraction, a decimal number, and a percent.

BIG IDEAS
This learning activity focuses on the following big ideas:

Quantity: Students compare fractional quantities expressed as a fraction, a decimal number, and a percent.

Relationships: Students investigate the relationships among fractions, decimal numbers, and percents by determining equivalent representations (e.g., $\frac{3}{4} = 0.75 = 75\%$), and by comparing numbers (e.g., $\frac{4}{5}$ is greater than 75%)

Representation: Students learn that fractional quantities can be expressed as fractions, decimal numbers, and percents, and that these number forms can represent the equivalent quantities (e.g., $\frac{1}{2}$, 0.5, 50%).

Proportional reasoning: This learning activity provides opportunities for students to use proportional reasoning as they determine equivalent fractions, decimal numbers, and percents.

CURRICULUM EXPECTATIONS
This learning activity addresses the following specific expectations. Students will:

• represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);

• represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, drawings, number lines, calculators) and using standard fractional notation;

• determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents (e.g., use a $10 \times 10$ grid to show that $\frac{1}{4} = 0.25$ or 25%).
These specific expectations contribute to the development of the following overall expectations.

Students will:
• read, represent, compare, and order whole numbers to 1,000,000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;
• demonstrate an understanding of relationships involving percent, ratio, and unit rate.

ABOUT THE LEARNING ACTIVITY

MATERIALS
• Dec6.BLM1: The Contest (1 copy per student)
• sheets of chart paper or large sheets of newsprint (1 sheet per pair of students)
• markers (a few per pair of students)
• Dec6.BLM2: 10 x 10 Grids (several copies for the class)
• calculators
• a variety of manipulatives for representing fractions, decimal numbers, and percents (e.g., base ten blocks, counters, fraction circles)
• sheets of paper or math journals (1 per student)
• Dec6.BLM3: Fractions and Percents in Ads (1 per student)

MATH LANGUAGE
• fraction • fractional number
• decimal number • equivalent
• percent • representation

INSTRUCTIONAL SEQUENCING
Before starting this learning activity, students should have had experiences exploring the meaning of “hundredth” and representing hundredths as fractions and decimal numbers. They should also have been introduced to basic ideas about percent (e.g., that percent means “per 100”) and should have had opportunities to represent percents on 10 x 10 grids. This learning activity allows students to recognize relationships among fractions, decimal numbers, and percent.

ABOUT THE MATH
This learning activity reinforces students’ understandings of fractions, decimal numbers, and percents, and of the connections among the three number forms. It helps students to understand that fractions, decimal numbers, and percents involve part-whole relationships:
• In a fraction, the whole is divided into equal parts.
• In a decimal number, the whole is divided into tenths, hundredths, thousandths, and so on.
• In a percent, the whole is comprised of 100 parts or 100%.

Fractions, decimal numbers, and percents can represent the same quantities (e.g., 1/2, 0.5, and 50% all represent “half”). By relating fractions, decimal numbers, and percents, students can determine equivalent forms (e.g., 4/5 = 0.8 = 80%) and can compare numbers (e.g., 3/5 is less than 80%).
GETTING STARTED

Tell students that they will be examining a difficult situation that faced a Grade 6 class in another school. Provide each student with a copy of Dec6.BLM1: The Contest, and read the page together. Discuss the situation, and remind students that the Grade 4 class understands fractions, that the Grade 5 class also understands decimal numbers, but that neither grade has studied percents. If students have questions about the number of students in each class, ask them to think about whether they need this information to solve the problem.

WORKING ON IT

Have students work with a partner. Provide each pair with markers and a sheet of chart paper or a large sheet of newsprint. Make available copies of Dec6.BLM2: 10 x 10 Grids, calculators, and a variety of manipulatives for representing fractions, decimal numbers, and percents (e.g., base ten blocks, counters, fraction circles). Encourage students to use these materials.

Remind students that their explanation must be clear enough for Grade 4 and Grade 5 students to understand, and suggest that they express their ideas using words, numbers, and/or diagrams.

As students work, observe their strategies, and ask them to fully explain their thinking. It may be necessary to encourage some students to add more detail to their explanations, or to try explaining their ideas in different ways.

Pose the following questions to students as they are working:

• "Do you agree that the Grade 6 class won the contest? How do you know?"
• "How can you clearly show that the Grade 6 class won the contest?"
• "How did you show your ideas so that Grade 4 students will understand your explanation?"

STRATEGIES STUDENTS MIGHT USE

USING 10 x 10 GRIDS

Students might shade 10 x 10 grids to represent and compare 4/5, 0.75, and 90%.
USING MANIPULATIVES OR DIAGRAMS
Students might use manipulatives (e.g., fraction circles) or diagrams to represent and compare the numbers.

![Fraction Circles Diagram]

USING NUMBER LINES
Students might compare the numbers by locating them on number lines.

![Number Line Diagram]

THINKING OF THE WHOLE AS 100
Students might think of the numbers in terms of a whole of 100. (Although 100 is not a realistic class size, it provides a common denominator for the fractional numbers from all three classes.)

Grade 4: \( \frac{4}{5} \) of 100 students is 80 students.
Grade 5: 0.75 of 100 students is 75 students.
Grade 6: 90\% of 100 students is 90 students.

REPRESENTING THE NUMBERS AS FRACTIONS, DECIMAL NUMBERS, AND PERCENTS
Students might represent the numbers in all three forms in order to compare them.
Reconvene the class and ask a few pairs of students to present their work to the class. Select pairs who used various approaches so that students can observe and discuss different ways of showing relationships among fractions, decimal numbers, and percents.

Focus the discussion on the concept that fractions, decimal numbers, and percents are all different ways to represent fractional numbers. Emphasize the idea that all three number forms represent parts of a whole – 5 fifths represents the whole in 4/5; 100 hundredths represents the whole in 0.75; and 100% represents the whole in 90%. Record “4/5”, “0.75”, and “90%” on the board, and ask questions such as the following:

- “What does 4/5 mean? What does 5 as the denominator mean? What does 4 as the numerator mean? Does the fraction mean that there are only 5 students in the class, and that 4 students donated to the food bank? How many fifths would there be if the entire class donated to the food bank?”
- “What does 0.75 mean? Does the decimal number mean that there were 100 students in the class? How many hundredths would there be if the entire class donated to the food bank?”
- “What does 90% mean? Does the percent mean that there were 100 students in the class? If the entire class donated to the food bank, what percent would that be?”

Extend students’ thinking by asking the following questions. Encourage students to use 10×10 grids and manipulatives (e.g., base ten blocks, counters, fraction circles) to explain their answers.

- “What fraction would the Grade 4 class need in order to win? How can you figure that out?”
- “What decimal number would the Grade 5 class need in order to win? How do you know?”
- “Which class had the lowest portion of students participating? How do you know?”
- “Suppose the Grade 5 and the Grade 6 classes tied with the Grade 4 class. What decimal number and what percent are equal to 4/5?”
- “Suppose the Grade 4 and the Grade 6 classes tied with the Grade 5 class? What fraction and what percent are equal to 0.75?”
- “Suppose the Grade 4 and the Grade 5 classes tied with the Grade 6 class? What fraction and what decimal number are equal to 90%?”

## Grade 6 Learning Activity: The Contest

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Grade 4: 4/5</th>
<th>Grade 5: 0.75</th>
<th>Grade 6: 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal Number</td>
<td>0.80</td>
<td>0.75</td>
<td>0.90</td>
</tr>
<tr>
<td>Percent</td>
<td>80%</td>
<td>75%</td>
<td>90%</td>
</tr>
</tbody>
</table>

REFLECTING AND CONNECTING

Reconvene the class and ask a few pairs of students to present their work to the class. Select pairs who used various approaches so that students can observe and discuss different ways of showing relationships among fractions, decimal numbers, and percents.
To conclude the learning activity, have students write a response to the following question on a sheet of paper or in their math journals:

“What portion of the students in each class did not donate to the food bank? Express the portion of students who did not participate in different ways. Use diagrams to support your explanation.”

**ADAPTATIONS/EXTENSIONS**

In this learning activity, students need to understand the meaning of fractions, decimal numbers, and percents. If students have difficulty dealing with all three number forms at the same time, provide a problem in which students need to compare numbers expressed in one form (i.e., only fractions, or only decimal numbers, or only percents). Encourage students to use materials such as 10 × 10 grids, fraction models, and base ten blocks to help them compare the numbers. As well, carefully consider student groupings. Place struggling students with classmates who will support them as they participate in and learn from the activity.

To extend the learning activity, provide students with the number of students in each class:
- 25 students in the Grade 4 class
- 28 students in the Grade 5 class
- 30 students in the Grade 6 class

Have students determine the number of students in each class who did and did not donate to the food bank.

**ASSESSMENT**

Observe students to assess how well they:
- represent fractions, decimal numbers, and percents using diagrams and manipulatives;
- explain that all three number forms (fractions, decimal numbers, percents) represent fractional parts of a whole;
- explain relationships among fractions, decimal numbers, and percents;
- compare fractions, decimal numbers, and percents.

Use the assessment information to help you make decisions about subsequent learning activities that will further students’ understanding of the relationships among fractions, decimal numbers, and percents.

**HOME CONNECTION**

Send home copies of Decimals, BLM3: Fractions and Percents in Ads. This letter encourages parents/guardians to discuss with their children the meaning of fractions and percents in sales ads.
LEARNING CONNECTION 1
Number Clothes Line

MATERIALS
• string or cord (approximately 2 to 3 m long)
• clothes pegs
• number cards (e.g., index cards) labelled “0” and “1”
• number cards (e.g., index cards) with a fraction, decimal number, or percent written on each card. (Sample numbers: 1/2, 2/3, 3/4, 9/10, 0.2, 0.33, 0.45, 0.59, 0.8, 1%, 10%, 25%, 28%, 95%, 100%)

Suspend a string or a cord in the classroom, and explain to students that it represents a number line. Use clothes pegs to attach number cards labelled “0” and “1” at opposite ends of the string. Show one of the number cards (fraction, decimal number, or percent), and ask students to discuss with a partner the position of the number on the line relative to 0 and 1. Ask a few students to explain where the number should be placed on the line. When the class agrees on an appropriate spot, attach the number card to the line using a clothes peg.

Provide each pair of students with a number card. Ask partners to discuss where their card should be placed on the number line. Ask each pair, one at a time, to show their number card to the class and then attach the card to the number line. Have students explain their rationale for positioning the card in its spot. Remind students that they need to determine the position of their number in relation to all numbers that have already been attached to the number line.

After all cards have been attached to the number line, ask students to examine the order of the numbers to ensure that they are arranged from least to greatest value. Have students justify any adjustments they would like to make.

This activity could be done in one day, or it could occur over several days by selecting a number card each day and discussing its position on the number line.

LEARNING CONNECTION 2
Fraction-Decimal-Percent Connectors

MATERIALS
• cards cut from Dec6.BLM4: Fraction-Decimal-Percent Connector Cards
• Dec6.BLM5: Fill-in-the-Blank Connector Cards (2 copies per pair of students)

Give a card from Dec6.BLM4: Fraction-Decimal-Percent Connector Cards to each student. Ask all students to stand. Begin by having the student with the starred card read his or her question, and then sit down. The student with the correct answer on his or her card reads the response, and then asks the question on his or her card and sits down. The activity continues until a student reads the last card and sits down. (The answer to the question on the last card is 3/4, which is the number on the first card. Extra cards can be created within the chain of cards as needed.)
The activity can be repeated after collecting and shuffling the cards, and redistributing the cards to students. Students might enjoy the challenge of improving the time it takes to complete the circuit.

Students can work with a partner to make their own cards using copies of Dec6.BLM5: Fill-in-the-Blank Connector Cards. Repeat the activity using the student-made cards.

**LEARNING CONNECTION 3**

**Equivalent Number Triplets**

**MATERIALS**
- Dec6.BLM6: Equivalent Number Triplets (1 per pair of students)
- index cards (30 per pair of students)
- Dec6.BLM7: Find the Equivalent Number Triplets (1 per pair of students)

This game reinforces the concept that a fractional number can be represented as a fraction, decimal number, or percent.

Have students prepare game cards. Provide each pair of students with a copy of Dec6.BLM6: Equivalent Number Triplets and 30 index cards. Have pairs record (or glue cut-outs of) each number from Dec6.BLM6: Equivalent Number Triplets onto a separate index card.

Explain that the game involves collecting sets of three number cards showing an equivalent fraction, decimal number, and percent. Explain that the goal of the game is to be the first player to collect two sets of cards.

To begin, one player shuffles the cards and deals six cards to each player. The remaining cards are placed face down in a pile between the two players. Players take turns drawing a card from the pile. If a player chooses to keep the card drawn from the pile, he or she must discard a card from his or her hand and place it face down at the bottom of the pile.

The game continues until one player collects two three-card sets.

As an extension, have pairs of students complete Dec6.BLM7: Find the Equivalent Number Triplets. Tell students that they are given a fraction, a decimal number, or a percent for each set of numbers, and that they must record the other two equivalent representations. After students complete the blackline master, they can prepare a set of game cards using the numbers on the worksheet.
LEARNING CONNECTION 4
Concentration

MATERIALS
• Dec6.BLM6: Equivalent Number Triplets (1 per pair of students)
• index cards (30 per pair of students)

Have students prepare game cards. Provide each pair of students with a copy of Dec6.BLM6: Equivalent Number Triplets and 30 index cards. Have pairs record (or glue cut-outs of) each of the numbers from Dec6.BLM6: Equivalent Number Triplets onto a separate index card. (Cards may have already been prepared for the previous learning connection.)

To play the game in pairs, one player shuffles the cards and places them face down in an array. Then the second player flips over three cards, trying to find cards that show an equivalent fraction, decimal number, and percent. If the player reveals a set of equivalent numbers, he or she keeps the set of cards and takes another turn. If the player does not find a complete set of matching cards, he or she flips the cards face down again, and the first player takes a turn.

The player with the most sets of cards at the end of the game wins.

LEARNING CONNECTION 5
The Greater Decimal Number

MATERIALS
• spinners made with Dec6.BLM8: 0-9 Spinner, a paper clip, and a pencil (1 per pair of students)
• sheets of paper (1 per student)
• manipulatives for representing decimal numbers (e.g., base ten blocks, metre sticks)

Provide each pair of students with the materials needed to make a spinner (a copy of Dec6.BLM8: 0-9 Spinner, a pencil, and a paper clip). Each player begins by drawing seven spaces for a “blank” number, such as the following, on a sheet of paper:

    _____ _____ _____ . _____ _____ _____

Players take turns spinning the spinner and recording the numeral shown on the spinner in one of the spaces of the number. Once a numeral has been recorded, players may not erase it or record it in a different space. When players have filled all seven spaces in their number, they compare them to determine which player has the greater number. Players may use manipulatives (e.g., base ten blocks, metre sticks) to help them compare the numbers. The player with the greater number wins the game.

After students have played the game a few times, discuss the strategies they used to help them create the greatest possible number.
eWORKSHOP CONNECTION
Visit www.eworkshop.on.ca for other instructional activities that focus on decimal concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Decimal Numbers (4 to 6)”, and then click the number to the right of it.
The Contest

The situation

Ms. MacIntosh’s Grade 6 class is holding a contest to encourage students in the school to donate to the local food bank. The class with the most students that contribute wins a pizza party.

All of the junior-grade classes have decided to enter. The Grade 6 students know that each class has a different number of students, so to be fair, they have asked each class to report the number of students contributing to the food bank as a portion of the whole class. The class with the greatest portion will win.

The results

• The Grade 4 class is learning about fractions and has reported that 4/5 of their students contributed to the food bank.
• The Grade 5 class is learning about decimal numbers and has reported that 0.75 of their students contributed to the food bank.
• The Grade 6 class is learning about percents and has reported that 90% of their students contributed to the food bank.

The Grade 6 students have decided that their own class is the winner. But they are concerned that the other classes might think that the decision is incorrect and that the Grade 6 class is dishonest.

Your challenge

First, determine whether the Grade 6 students’ decision is correct. Then prepare an explanation for the Grade 4 and Grade 5 classes that will help these students understand the decision, so they won’t think that the older students are cheating. You may use any classroom materials and manipulatives to help you explain your ideas. Show your explanation clearly on the paper given to you.
10×10 Grids
Fractions and Percents in Ads

Dear Parent/Guardian:

We are learning about fractions and percents. Fractions and percents are often used in newspapers and flyer ads to describe sales.

With your child, find ads with fractions and percents. Ask your child to explain the discount described by the fraction or percent.

Ask other questions such as the following:
• “What would you pay for an item that is regularly $28, if it is 1/2 price?”
• “What does it mean if items are ‘up to 25% off’?”
• “How could you figure out, in your head, what you would save if an item that regularly costs $35 is discounted by 10%?”
• “Which store(s) is offering the best discount?”
• “Which store(s) is offering the least discount?”
• “What would this ad say if a fraction were used instead of a percent?”
• “Do most stores use fractions or percents in their ads? Why do you think they do?”

Thank you for discussing fractions and percents with your child.
### Fraction-Decimal-Percent Connector Cards

| I have $\frac{3}{4}$.  
<table>
<thead>
<tr>
<th>Who has a decimal equivalent to $\frac{1}{2}$?</th>
</tr>
</thead>
</table>
| I have 0.5.  
 | Who has a percent equivalent to $\frac{1}{5}$? |
| I have 20%.  
 | Who has a decimal equivalent to $\frac{1}{4}$? |
| I have 0.25.  
 | Who has a fraction equivalent to 60%? |

| I have $\frac{3}{5}$.  
<table>
<thead>
<tr>
<th>Who has a percent equivalent to $\frac{1}{2}$?</th>
</tr>
</thead>
</table>
| I have 25%.  
 | Who has a fraction equivalent to 90%? |
| I have $\frac{9}{10}$.  
 | Who has a decimal equivalent to 45%? |
| I have 0.45.  
 | Who has a percent equivalent to 0.56? |

| I have 56%.  
<table>
<thead>
<tr>
<th>Who has a fraction equivalent to 20%?</th>
</tr>
</thead>
</table>
| I have $\frac{1}{5}$.  
 | Who has a decimal equivalent to $\frac{3}{5}$? |
| I have 0.6.  
 | Who has a percent equivalent to $\frac{1}{10}$? |
| I have 10%.  
 | Who has a fraction equivalent to 11%? |

| I have $\frac{11}{100}$.  
<table>
<thead>
<tr>
<th>Who has a decimal equivalent to $\frac{7}{10}$?</th>
</tr>
</thead>
</table>
| I have 0.7.  
 | Who has a percent equivalent to $\frac{1}{2}$? |
| I have 50%.  
 | Who has a fraction equivalent to 80%? |
| I have $\frac{4}{5}$.  
<p>| Who has a decimal equivalent to $\frac{1}{5}$? |</p>
<table>
<thead>
<tr>
<th>I have 0.2. Who has a percent equivalent to 0.7?</th>
<th>I have 7%. Who has a fraction equivalent to 0.07?</th>
<th>I have $\frac{7}{100}$. Who has a percent equivalent to 0.3?</th>
<th>I have 30%. Who has a decimal equivalent to 7%?</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have 0.07. Who has a fraction equivalent to 30%?</td>
<td>I have $\frac{3}{10}$. Who has a decimal equivalent to 99%?</td>
<td>I have 0.99. Who has a percent equivalent to 0.04?</td>
<td>I have 4%. Who has a fraction equivalent to 0.25?</td>
</tr>
<tr>
<td>I have $\frac{1}{4}$. Who has a percent equivalent to $\frac{4}{5}$?</td>
<td>I have 80%. Who has a decimal equivalent to 4%?</td>
<td>I have 0.04. Who has a percent equivalent to $\frac{2}{10}$?</td>
<td>I have 90%. Who has a fraction equivalent to 0.75?</td>
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</tbody>
</table>
## Fill-in-the-Blank Connector Cards

<table>
<thead>
<tr>
<th>I have ________.*</th>
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<tbody>
<tr>
<td>Who has</td>
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<td>_________________?</td>
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<td>________________?</td>
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<tr>
<td>Who has</td>
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<tbody>
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<td>Who has</td>
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## Equivalent Number Triplets

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal Number</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>0.5</td>
<td>50%</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>0.75</td>
<td>75%</td>
</tr>
<tr>
<td>$\frac{1}{10}$</td>
<td>0.1</td>
<td>10%</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
<td>0.2</td>
<td>20%</td>
</tr>
<tr>
<td>$\frac{3}{10}$</td>
<td>0.3</td>
<td>30%</td>
</tr>
<tr>
<td>$\frac{2}{5}$</td>
<td>0.4</td>
<td>40%</td>
</tr>
<tr>
<td>$\frac{3}{5}$</td>
<td>0.6</td>
<td>60%</td>
</tr>
<tr>
<td>$\frac{7}{10}$</td>
<td>0.7</td>
<td>70%</td>
</tr>
<tr>
<td>$\frac{4}{5}$</td>
<td>0.8</td>
<td>80%</td>
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</table>
Find the Equivalent Number Triplets

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0–9 Spinner
Make a spinner using this page, a paper clip, and a pencil.